Measuring Connectedness using Large TVP-VAR Models

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Introduction

- Diebold and Yilmaz (2014) used the connectedness index (DYCI) framework to show how volatility shocks to major EU and US financial institution stocks led to substantial connectedness across Europe and the Atlantic.
- DYCI framework relies on rolling sample windows estimation of VARs to obtain dynamic connectedness measures.
- The resulting dynamic total connectedness indices possess extra persistence imposed by the fixed-length rolling windows estimation and reflect the simultaneous influence of several episodes that are included in the sample window.
- We estimate a TVP-VAR model for the same set of variables to remedy these shortcomings

A Natural Financial/Economic Connectedness Question

What fraction of the H-step-ahead prediction-error of variable i is due to shocks in variable j, $j \neq i$?

Off-diagonal elements of the variance decomposition matrix,
$d_{ij}^{H}, j \neq i$

Variance Decompositions as Weighted, Directed Network

Variance Decomposition / Connectedness Table

	$\mathbf{x_1}$	$\mathbf{x_2}$	...	$\mathbf{x_N}$	From Others
$\mathbf{x_1}$	d^H_{11}	d^H_{12}	$\cdots$	d^H_{1N}	$\sum_{j \neq 1} d^H_{1j}$
$\mathbf{x_2}$	d^H_{21}	d^H_{22}	$\cdots$	d^H_{2N}	$\sum_{j \neq 2} d^H_{2j}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$\mathbf{x_N}$	d^H_{N1}	d^H_{N2}	$\cdots$	d^H_{NN}	$\sum_{j \neq N} d^H_{Nj}$
To Others	$\sum_{i \neq 1} d^H_{i1}$	$\sum_{i \neq 2} d^H_{i2}$	$\cdots$	$\sum_{i \neq N} d^H_{iN}$	$\sum_{i \neq j} d^H_{ij}$

Total directional connect. "from," $C^H_{i \leftarrow \bullet} = \sum_{\substack{j=1 \\ j \neq i}}^{N} d^H_{ij}$: "from-degrees"

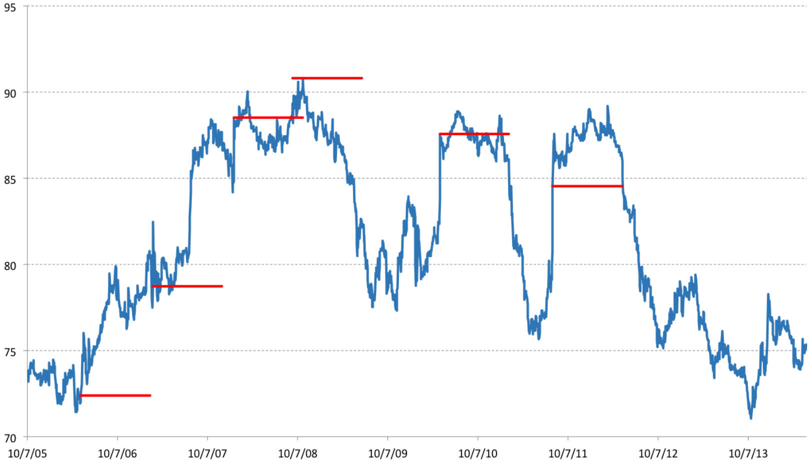
Total directional connect. "to," $C^H_{\bullet \leftarrow j} = \sum_{\substack{i=1 \\ i \neq j}}^{N} d^H_{ij}$: "to-degrees"

Systemwide connectedness, $C^H = \frac{1}{N} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} d^H_{ij}$: mean degree

Connectedness Measures

- Pairwise Directional: $C^H_{j\leftarrow i} = d^H_{ij}$
- Net Pairwise Directional: $C^H_{ij} = C^H_{j\leftarrow i} - C^H_{i\leftarrow j}$
- Total Directional:
  - **From others** to i: $C^H_{i\leftarrow\bullet} = \sum_{\substack{\mathbf{j=1} \\ j\neq i}}^{N} d^H_{ij}$
  - From j **To others**: $C^H_{\bullet\leftarrow j} = \sum_{\substack{\mathbf{i=1} \\ i\neq j}}^{N} d^H_{ij}$
- Net Total Directional: $C^H_i = C^H_{\bullet\leftarrow i} - C^H_{i\leftarrow\bullet}$
- Total Connectedness: $C^H = \frac{1}{N}\sum_{\substack{i,j=1 \\ i\neq j}}^{N} d^H_{ij}$

Total Connectedness with 200-day rolling windows



This Paper

- Dynamic total connectedness indices obtained from VARs estimated over rolling sample windows and the large TVP-VAR model differ substantially.
- While the index obtained from rolling-windows approach jumps very little during (some) important crisis moments, the one obtained from the large TVP-VAR model shows more pronounced jumps during those crisis moments.
- Connectedness index obtained from TVP-VAR model declines as the impact of the volatility shock on stock return volatilities diminish.
- The rolling-sample window based index, on the other hand, stays high as long as the data pertaining to the crisis moment is kept within the rolling-sample window.

TVP-VAR Model

- We specify the following time-varying parameter VAR(p) model

$$y_t = \varphi_{0t} + \Phi_{1t} y_{t-1} + ... + \Phi_{pt} y_{t-p} + \varepsilon_t,$$

  where $\varepsilon_t \sim N(0, \Sigma_t)$

- Consider the $K \times 1$ vector of coefficients $\beta_t = vec\left(\left[\varphi'_{0t}, \Phi'_{1t}, ..., \Phi'_{pt}\right]'\right)$ along with the $M \times K$ vector $x_t = I \otimes \left[1, y'_{t-1}, ..., y'_{t-p}\right]$.

TVP-VAR Model

- When limited information is available about the exact nature of parameter changes, the vector of coefficients β_t are usually allowed to follow a random walk,
- So the TVP-VAR can be represented as

$$
\begin{aligned}
y_t &= \beta_t x_t + \varepsilon_t, \\
\beta_t &= \beta_{t-1} + \eta_t,
\end{aligned}
$$

where $\eta_t \sim N(0, \Omega_t)$.

Sequential Bayesian inference for TVP-VARs

- Initial condition for the model is given by

$$
\begin{aligned}
\beta_0 &\sim N(m_0, C_0) \\
\Sigma_0 &\sim iW(S_0, n_0)
\end{aligned}
$$

- Time t priors is given by

$$
\begin{aligned}
\beta_t | D_{t-1} &\sim N\left(m_{t|t-1}, C_{t|t-1}\right) \\
\Sigma_t | D_{t-1} &\sim iW\left(S_{t|t-1}, n_{t|t-1}\right)
\end{aligned}
$$

  where $m_{t|t-1} = m_{t-1}$, $C_{t|t-1} = \frac{1}{\lambda} C_{t-1}$, $S_{t|t-1} = S_{t-1}$, and $n_{t|t-1} = \delta n_{t-1}$.

- Variance discounting (or forgetting) factors $\lambda, \delta \in (0, 1]$ (see Koop and Korobilis, 2013, JoE).

Sequential Bayesian inference for TVP-VARs

- Posterior of $\Sigma_{t|t}$

$$\Sigma_t | D_t \sim iW\left(S_t, n_t\right)$$

where $n_t = \delta n_{t-1} + 1$ and $S_{t|t} =$
$\left(1 - a_t\right) S_{t-1|t-1} + a_t \left[S_{t-1|t-1}^{1/2} Q_{t-1}^{-1/2} \left(\varepsilon_t \varepsilon_t'\right) Q_{t-1}^{-1/2} S_{t-1|t-1}^{1/2}\right]$,
with $a_t = n_t^{-1}$.
In this formulation, ε_t is replaced with the one-step ahead prediction error $\widetilde{\varepsilon}_{t|t-1} = y_t - m_{t|t-1} x_t$.

- Posterior of $\beta_{t|t}$

$$\beta_t | \Sigma_t, D_t \sim N\left(m_t, C_t\right)$$

where $m_t = m_{t|t-1} + C_{t|t-1} x_t V_t^{-1} \widetilde{\varepsilon}_t$ and
$C_t = C_{t|t-1} - C_{t|t-1} x_t' V_t^{-1} x_t C_{t|t-1}$, with $\widetilde{\varepsilon}_t = y_t - x_t m_{t|t-1}$ the prediction error and $V_t = x_t C_{t|t-1} x_t' + \Sigma_t$ its covariance matrix.

A Simple Adaptive Estimation Algorithm for TVP-VARs

- In the case of Ω we set

$$\Omega_t = \left(\lambda^{-1} - 1\right) var\left(\beta_t | \mathcal{D}_{t-1}\right),$$

where at time t the quantity $var\left(\beta_t | \mathcal{D}_{t-1}\right)$ is readily available from the Kalman filter.

- The quantity $0 < \lambda \leq 1$ is a forgetting factor which controls how fast the time-variation occurs in β_t.
- Thus a lower λ implies that fewer observations are used for estimation of β_t, hence older data are forgotten in a faster rate and β_t can vary substantially from one period to the next. In the extreme case $\lambda = 1$ we can see that $\Omega_t = 0$ in which case $\beta_t = \beta_{t-1}$ for all t, i.e. β_t becomes a constant parameter.
- Here we select a single forgetting factor λ which we actually update according to the formula:

$$\lambda_t = \underline{\lambda} + \left(1 - \underline{\lambda}\right) \exp\left(-0.5 diag\left(\widetilde{\varepsilon}_t' V_t^{-1} \widetilde{\varepsilon}_t\right)\right),$$

Connectedness of Major U.S. & EU Financial Institutions

- Variable of interest: Log daily range volatilities for stock returns
  - 17 U.S. financial institutions
  - 18 European financial institutions: Belgium (2), France (3), Germany (2), Italy (2), Netherlands (1), Spain (2), Switzerland (2), UK (4).
- Data coverage: 1/2/2004 – 5/30/2014 (first 250 days used as training sample)
- Approximating model: **VAR**? Structural DSGE?
- Estimation: Classical? **Bayesian**? Hybrid?
- Time-varying connectedness: Rolling estimation? **Smooth TVP's?** Regime switching?
- Identification of variance decomp.: Cholesky? **Generalized**? SVAR?

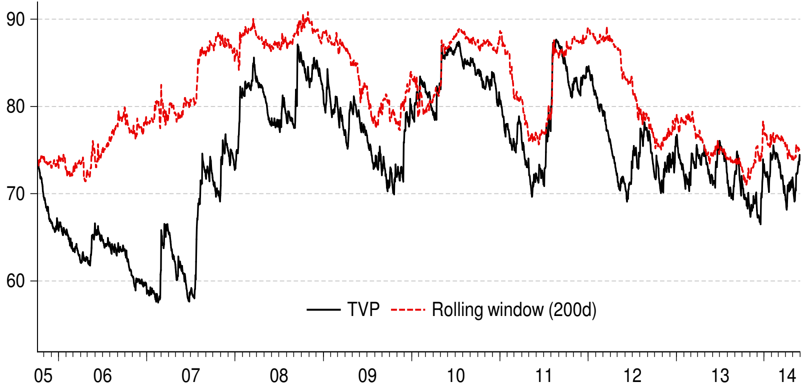
U.S. Financial Institution Detail

Institution	Ticker	Market Cap. 12/29/06	Market Cap. 5/30/14	Assets 3/31/14
JP Morgan Chase	JPM	169	210	2,477
Bank of America	BAC	241	159	2,150
Citigroup	C	274	145	1,895
Wells Fargo	WFC	121	267	1,547
Goldman Sachs	GS	86	71	916
Morgan Stanley	MS	85	61	831
US Bancorp	USB	64	77	371
Bank NY Mellon	BK	30	39	368
PNC Financial	PNC	22	46	323
American Express	AXP	74	97	151
Fannie Mae	FNM	59	1.3	–
Freddie Mac	FRE	47	0.9	–
AIG	AIG	187	4	547
Bear Stearns	BSC	19	Acquired by JPM	3/17/08
Lehman Brothers	LEH	41	Bankruptcy	9/15/08
Merrill Lynch	MER	82	Acquired by BAC	9/15/08
Wachovia Bank	WB	115	Acquired by WFC	10/3/08

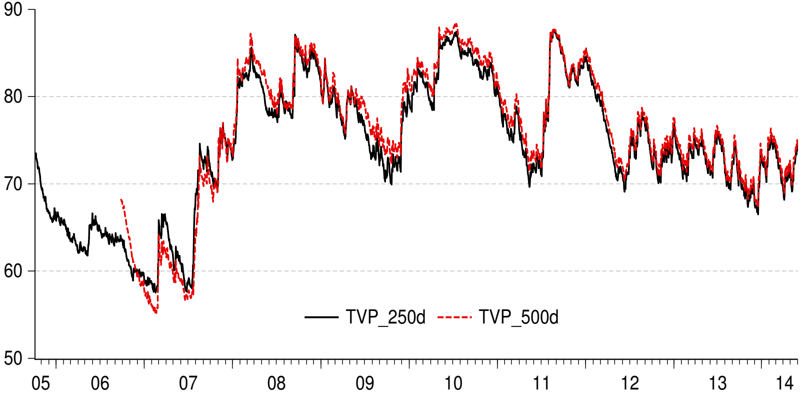
EU Financial Institution Detail

Institution	Ticker	Country	Market Cap. 12/29/06	Market Cap. 5/30/14	Assets 3/31/14
Dexia	DEX	Belgium	31	0.1	473
KBC	KBC		45	25	339
Deutsche Bank	DBK	Germany	70	41	2,254
Commerzbank	CBK		25	18	791
BNP Paribas	BNP	France	101	87	2,593
Societe Generale	GLE		79	46	1,743
Credit Agricole	ACA		63	39	2,139
Unicredit Group	UCG	Italy	91	51	1,159
Intesa San Paolo	ISP		46	52	861
ING Bank	ING	Netherlands	98	54	1,306
Bank Santander	SAN	Spain	117	121	1,610
BBVA	BBVA		85	76	825
UBS	UBS	Switzerland	128	77	993
Credit Suisse Group	CSG		85	48	1,111
HSBC	HSBA	UK	211	201	2,758
Barclays	BARC		93	68	2,272
Royal B. Scotland	RBS		123	36	1,708
Lloyds Bank	LLOY		63	93	1,405

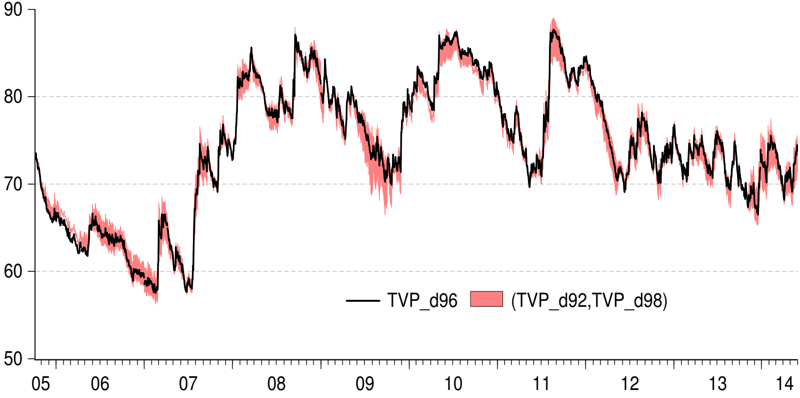
Total Connectedness - TVP-VAR vs 200-day rolling window



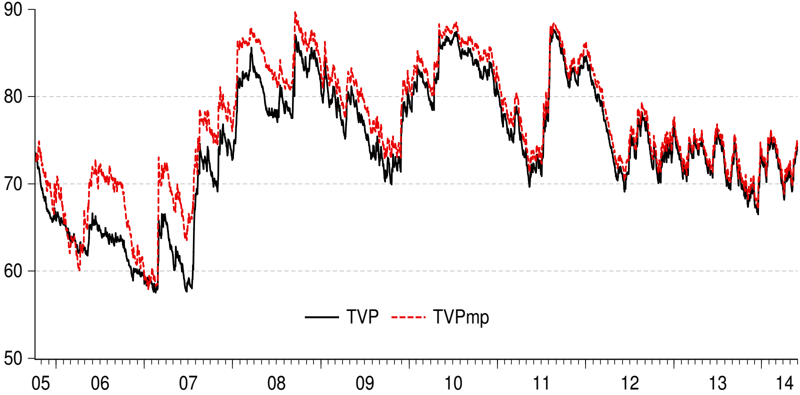
Training sample size - 250d vs 500d



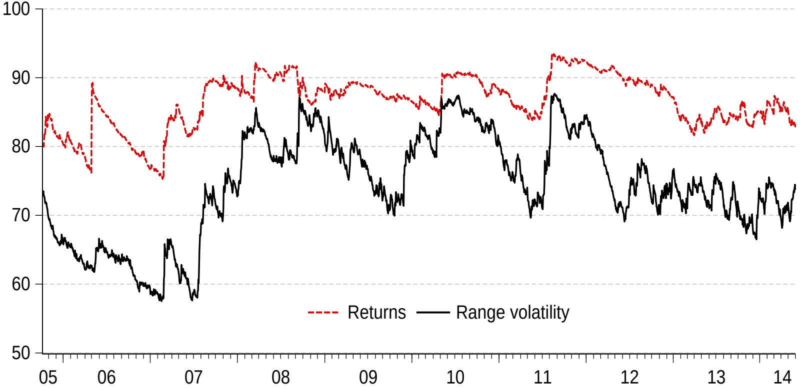
Decay factor δ



TVP-VAR with Training Sample vs Minnesota Prior



TVP-VAR for volatility vs returns

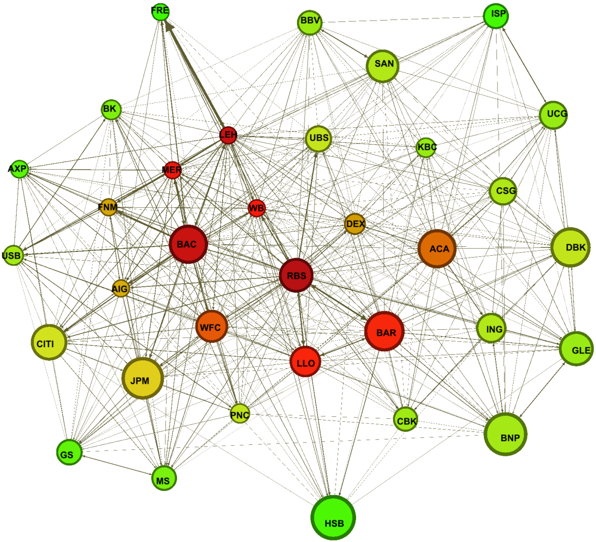


A Final Choice: Graphical Display via "Spring Graphs"

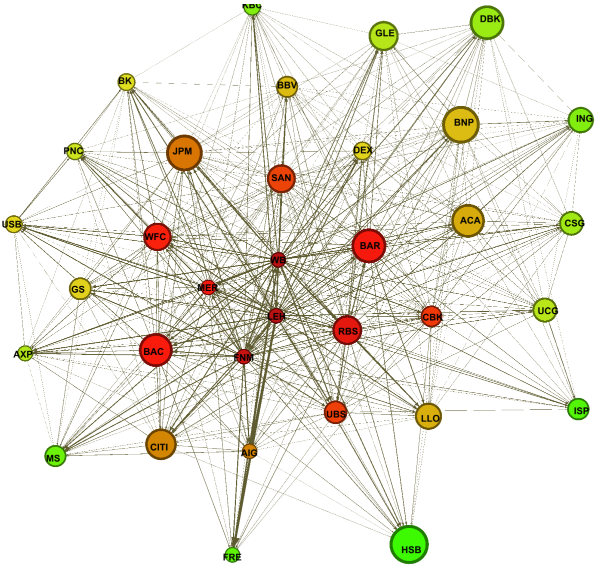
- Node size: Asset size
- Node color: Total directional connectedness "to others"

- Node location: Average pairwise directional connectedness (Equilibrium of repelling and attracting forces, where (1) nodes repel each other, but (2) edges attract the nodes they connect according to average pairwise directional connectedness "to" and "from.")
- Edge thickness: Average pairwise directional connectedness
- Edge arrow sizes: Pairwise directional "to" and "from" connectedness

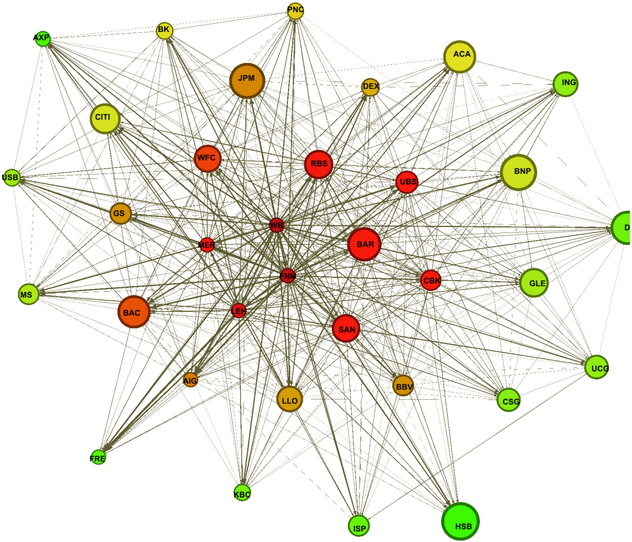
Pre-Lehman Crisis – September 4, 2008



Lehman Bankruptcy Announced – September 15, 2008



Three Days into the Lehman Crisis – September 18, 2008



Conclusions

- Rolling sample windows estimation was a major limitation of the DYCI framework
- The resulting dynamic total connectedness indices possess extra persistence and reflect confluence of several episodes
- In this paper we estimated a large TVP-VAR model EU and US financial institution stock return volatilities
- We show that the large TVP-VAR model estimation solves the excessive persistence problem found in dynamic connectedness measures as well as identifying the impact of each episode on volatility connectedness across financial institutions.
- In terms of "economic" fit the TVP-VAR model based CI performs much better than the rolling-windows based CI. We hope to show that it also performs better in out-of-sample forecasting.