Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Dynamic Factor Model with infinite-dimensional factor space: forecasting

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Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Outline

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Outline

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Outline

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Outline

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

General form of DFM

General form of the Large-Dimensional Dynamic Factor Model

$$x_{it} = \frac{c_{i1}(L)}{d_{i1}(L)}u_{1t} + \frac{c_{i2}(L)}{d_{i2}(L)}u_{2t} + \cdots + \frac{c_{iq}(L)}{d_{iq}(L)}u_{qt} + \xi_{it}, \quad i \in \mathbb{N}, \quad (1)$$

where

$$c_{if}(L) = c_{if,0}+c_{if,1}L+\ldots+c_{if,s_1}L^{s_1} \quad \text{and} \quad d_{if}(L) = d_{if,0}+d_{if,1}L+\ldots+d_{if,s_2}L^{s_2},$$

$\mathbf{u}_t = (u_{1t}\ u_{2t}\ \cdots\ u_{qt})'$ is a q-dimensional white noise, the variables ξ_{it} are idiosyncratic components.

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Finite dimension

If the common components

$$\chi_{it} = \frac{c_{i1}(L)}{d_{i1}(L)} u_{1t} + \frac{c_{i2}(L)}{d_{i2}(L)} u_{2t} + \cdots + \frac{c_{iq}(L)}{d_{iq}(L)} u_{qt}, \quad i \in \mathbb{N},$$

span a **finite-dimensional vector space**, then (1) can be rewritten in the **static form**

$$x_{it} = \lambda_{i1} F_{1t} + \lambda_{i2} F_{2t} + \cdots + \lambda_{ir} F_{rt} + \xi_{it}. \tag{2}$$

For example, if

$$\chi_{it} = \frac{a_{i1}}{1 - \alpha L} u_{1t} + a_{i2} u_{2t} + a_{i3} u_{2,t-1}, \tag{3}$$

then the model has a **static representation** with $r = 3$,

$$\lambda_{ij} = a_{ij}, \quad F_{1t} = (1 - \alpha L)^{-1} u_{1t}, \quad F_{2t} = u_{2t}, \quad F_{3t} = u_{2,t-1}.$$

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Finite dimension

$$x_{it} = \lambda_{i1}F_{1t} + \lambda_{i2}F_{2t} + \cdots + \lambda_{ir}F_{rt} + \xi_{it}. \tag{2}$$

The finite-dimension assumption or, equivalently, the existence of the static representation (2), has been almost universally adopted in the recent literature (see in particular Stock and Watson, 2002a,b, Bai and Ng, 2002).

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Finite dimension

Under the **finite-dimension assumption**, the factors F_{jt} and the loadings λ_{ij} can be consistently estimated, as the number of variables and the number of observations for each variable tend to infinity, by means of the **principal components of the observables** x_{it}. The estimated factors are then used as predictors in forecasting the variables x_{it}.

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Infinite dimension

A motivation for studying the general model (1), without restrictions on the dimension of the factor space, as argued in FHLZ (2015), is that the finite-dimension assumption (2) rules out models as simple as

$$x_{it} = \frac{a_i}{1 - \alpha_i L} u_t + \xi_{it}, \tag{4}$$

i.e.

$$x_{it} = a_i(u_t + \alpha_i u_{t-1} + \alpha_i^2 u_{t-2} + \cdots) + \xi_{it},$$

for $i \in \mathbb{N}$, where u_t is a scalar white noise and the coefficients α_i are drawn from, say, the uniform distribution between -0.8 and 0.8. For, unless α_i takes a finite number of values as $i \in \mathbb{N}$, the stochastic variables $\chi_{it} = a_i(1 - \alpha_i L)^{-1} u_t$ span an **infinite-dimensional space.**

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Infinite dimension

On the other hand, as pointed out in Forni et al. (2015a, Journal of Econometrics), when a dataset is given, with finite *n* (number of variables) and *T* (number of observazions), **the static method might perform well even under misspecification,** i.e. even if the data were generated by a model not fulfilling the finite-dimension assumption.

In Forni et al. (2015b) the static and the dynamic methods have been applied to simulated data in several Monte Carlo experiments. A very short summary of the results is that (i) when the data are generated by infinite-dimensional models which generalize (4), the estimation of impulse-response functions and predictions obtained by the dynamic method are by far better than those obtained by the static method; (ii) when the data are generated by (2), still the dynamic method performs slightly better.

THE PRESENT PAPER COMPARES THESE METHODS USING MONTHLY MACROECONOMIC US DATA 1959-2014 (THE STOCK-WATSON DATASET).

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Previous literature comparing different factor models

—Boivin and Ng (2005), International Journal of Central Banking
—D'Agostino and Giannone (2012) Oxford Bulletin of Economics and Statistics
—Schumacher (2007), Journal of Forecasting
—Luciani (2014), International Journal of Forecasting

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

SW (time domain)
FHLR (frequency domain)

Prediction: Stock and Watson

Starting with (2) the forecasting equation is obtained projecting $x_{1,t+1}$ on the space spanned by

$$\mathbf{F}_t, \ \mathbf{F}_{t-1}, \ldots; \ x_{1t}, \ x_{1,t-1}, \ldots$$

Thus

$$x_{1,t+1|t} = \boldsymbol{\alpha}(L)\mathbf{F}_t + \beta(L)x_{1t}, \tag{5}$$

where lagged vales of x_{1t} account for possible autocorrelation (predictability) of the idiosyncratic component. Equation (5) is often used without lags for $\mathbf{F}_t$ or without the term including $x_{1,t}$:

$$x_{1,t+1|t} = \boldsymbol{\alpha}\mathbf{F}_t + \beta(L)x_{1t}, \quad x_{1,t+1|t} = \boldsymbol{\alpha}\mathbf{F}_t.$$

We refer to this predictor as SW, from Stock and Watson (2002).

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

SW (time domain)
FHLR (frequency domain)

Prediction: FHLR

FHLR. **(i)** The procedure starts with the estimation of the spectral density matrix of the x's, call it $\boldsymbol{\Sigma}^x(\theta)$.
(ii) q is determined by the Hallin and Liska Criterion.
(iii) The spectral density of the common components is estimated using the first q Brillinger principal components of the x's (by means of the frequency domain eigenvectors of $\boldsymbol{\Sigma}^x(\theta)$), call it $\boldsymbol{\Sigma}^\chi(\theta)$. **(iv)** $\boldsymbol{\Sigma}^\chi(\theta)$ and $\Sigma^\xi(\theta)$ are used to compute the autocovariance matrices of the common and idiosyncratic components:

$$\boldsymbol{\Gamma}_k^\chi, \quad \boldsymbol{\Gamma}_k^\xi, \quad k \in \mathbb{Z},$$

$$\boldsymbol{\Gamma}_k^\chi = \int_{-\pi}^{\pi} e^{ik\theta} \boldsymbol{\Sigma}^\chi(\theta) \, d\theta, \quad \boldsymbol{\Gamma}_k^\xi = \int_{-\pi}^{\pi} e^{ik\theta} \boldsymbol{\Sigma}^\xi(\theta) \, d\theta$$

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

SW (time domain)
FHLR (frequency domain)

Prediction: FHLR

(iv) Under the finite-dimension assumption, the covariances Γ_0^χ and Γ_0^ξ are employed to estimate a basis in the factor space by means of generalized principal components (the estimated variance of the idiosyncratic is taken into account):

$$G_{1t},\ G_{2t},\ \ldots,\ G_{rt}.$$

The covariances Γ_1^χ and Γ_1^ξ are employed to project $\chi_{1,t+1}$ on the factors:

$$\chi_{1,t+1|t} = \gamma \mathbf{G}_t$$

(a dynamically more complex version, allowing for lags of the factors, as in SW, can be obtained in the same way). Lastly, the idiosyncratic component is predicted by means of a unvariate model. This predictor is based on Forni, Hallin, Lippi and Reichlin (2000, 2005). We refer to it as FHLR.

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ. Blockwise autoregressive representation and the static form

Prediction: FHLZ

Let us go back to model (1), reported here for convenience:

$$x_{it} = \frac{c_{i1}(L)}{d_{i1}(L)} u_{1t} + \frac{c_{i2}(L)}{d_{i2}(L)} u_{2t} + \cdots + \frac{c_{iq}(L)}{d_{iq}(L)} u_{qt} + \xi_{it}, \quad i \in \mathbb{N},$$

where

$$c_{if}(L) = c_{if,0} + c_{if,1}L + \ldots + c_{if,s_1}L^{s_1} \quad \text{and} \quad d_{if}(L) = d_{if,0} + d_{if,1}L + \ldots + d_{if,s_2}L^{s_2},$$

with common components

$$\chi_{it} = \frac{c_{i1}(L)}{d_{i1}(L)} u_{1t} + \frac{c_{i2}(L)}{d_{i2}(L)} u_{2t} + \cdots + \frac{c_{iq}(L)}{d_{iq}(L)} u_{qt}.$$

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ. Blockwise autoregressive representation and the static form

Prediction: FHLZ. Preliminary

We start in the same way as in FHLR (2005), but now we have rates.
(i) We start with an estimated lag-window spectral density $\hat{\Sigma}_n^x(\theta)$. We assume that

$$\hat{\Sigma}_{ij}^x(\theta) \to \Sigma_{ij}^x(\theta)$$

in probability for $T \to \infty$, uniformly in θ, i and j, with rate $1/\sqrt{\rho_T}$, where $\rho_T = T^{2/3}/\log T$ (see Wu and Zaffaroni, 2015). This is the price paid to non parametric estimation of the spectrum of the x's.
(ii) Then we obtain an estimator for the spectral density of the χ's, $\hat{\Sigma}_n^{\chi}(\theta)$ (principal components in the frequency domain), and prove that

$$\hat{\Sigma}_{ij,n}^{\chi}(\theta) \to \Sigma_{ij}^{\chi}(\theta)$$

as T and n tend to infinity, in probability at the rate $\max(n^{-1/2}, \rho_T^{-1/2})$.

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ. Blockwise autoregressive representation and the static form

Prediction: FHLZ. Preliminary

(iii) Then we prove that

$$\hat{\gamma}_{js,k,n}^{\chi} = \int_{-\pi}^{\pi} \hat{\sigma}_{js,n}^{\chi}(\theta)e^{ik\theta}d\theta \rightarrow \int_{-\pi}^{\pi} \sigma_{js,n}^{\chi}(\theta)e^{ik\theta}d\theta = \gamma_{js,k}^{\chi}$$

in probability as T and n tend to infinity, at the same rate as above.
Here we move away from FHLR in that we do not assume finite dimension of the factor space.

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ. Blockwise autoregressive representation and the static form

FHLZ. Autoregressive representation for the common components

We have the vector

$$\boldsymbol{\chi}_t = (\chi_{1t} \; \chi_{2t} \; \cdots \; \chi_{nt}, \; \cdots)',$$

and we have estimated (we know) its spectral density and therefore its covariance function.

In principle we can estimate a VAR for $\boldsymbol{\chi}_t$. You may object: but if you could not estimate a VAR for $\mathbf{x}_t$ because of the large dimension, how do you think you can do it for $\boldsymbol{\chi}_t$?

The answer is: unlike $\mathbf{x}_t$, $\boldsymbol{\chi}_t$ is a singular vector, a large dimensional vector driven by a small number of shocks.

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ. Blockwise autoregressive representation and the static form

FHLZ. Consequences of the singularity of χ_t

Singular vectors have special properties, recently studied by Anderson and Deistler in a number of papers. An example. Let $n = 2$ and $q = 1$:

$$
\begin{aligned}
y_{1t} &= \alpha_1 w_t + \beta_1 w_{t-1} \\
y_{2t} &= \alpha_2 w_t + \beta_2 w_{t-1}
\end{aligned}
$$

We see that

$$
w_t = \frac{1}{\alpha_1 \beta_2 - \alpha_2 \beta_1} (\beta_2 y_{1t} - \beta_1 y_{2t}),
$$

so that

$$
\left(
\begin{array}{cc}
1 - \delta \beta_1 \beta_2 L & \delta \beta_1^2 L \\
-\delta \beta_2^2 & 1 + \delta \beta_1 \beta_2 L
\end{array}
\right)
\left(
\begin{array}{c}
y_{1t} \\
y_{2t}
\end{array}
\right)
=
\left(
\begin{array}{c}
\alpha_1 \\
\alpha_2
\end{array}
\right)
w_t
$$

where $\delta = 1/(\alpha_1 \beta_2 - \alpha_2 \beta_1)$. Note that the autoregressive representation exists if and only if $\alpha_1 \beta_2 - \alpha_2 \beta_1 \neq 0$, thus generically.

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ. Blockwise autoregressive representation and the static form

FHLZ. Consequences of the singularity of χ_t

Suppose now that the dimension of the vector is 4, with $q = 1$:

$$y_{it} = \alpha_i w_t + \beta_i w_{t-1}, \quad i = 1, 2, 3, 4$$

You see that this vector has, generically, the autoregressive representation

$$\begin{pmatrix} 1 - a_{11}L & a_{12}L & 0 & 0 \\ a_{21}L & 1 - a_{22}L & 0 & 0 \\ 0 & 0 & 1 - a_{31}L & a_{32}L \\ 0 & 0 & a_{41}L & 1 - a_{42}L \end{pmatrix} \begin{pmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} w_t$$

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ. Blockwise autoregressive representation and the static form

FHLZ: Additional assumption

A. Current and past values of any $(q+1)$-tuple

$$(\chi_{i_1,t} \ \chi_{i_2,t} \ \cdots \ \chi_{i_{q+1},t})$$

span the same space spanned by current and past values of all the χ_{it}.
Let me illustrate this by an example. Assume $q = 1$ and

$$
\begin{aligned}
\chi_{1t} &= u_{t-1} \\
\chi_{2t} &= u_{t-1} \\
\chi_{jt} &= u_t, \ \text{for } j \geq 3
\end{aligned}
$$

Then Assumption A does not hold. Thus (ii) rules out cases like this.
On the other hand, if

$$\chi_{it} = \alpha_i u_t + \beta_i u_{t-1},$$

then generically A holds.

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ. Blockwise autoregressive representation and the static form

FHLZ: AR representation for the common components

Assume for convenience that $n = (q + 1)m$. Under Assumptions (i) and (ii) we obtain the representation

$$\begin{pmatrix} A^1(L) & 0 & \cdots & 0 \\ 0 & A^2(L) & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & A^m(L) \end{pmatrix} \begin{pmatrix} \chi_t^1 \\ \chi_t^2 \\ \vdots \\ \chi_t^m \end{pmatrix} = \begin{pmatrix} R^1 \\ R^2 \\ \vdots \\ R^m \end{pmatrix} \mathbf{v}_t$$

$$A_n(L)\chi_{nt} = R_n\mathbf{v}_t$$

where the vectors χ_t^k are non overlapping $(q + 1)$-dimensional selections, the blocks $A^k(L)$ are $(q + 1) \times (q + 1)$, the matrix R_n is $n \times q$ and $\mathbf{v}_t$ is a q dimensional white noise. The matrices $A^k(L)$ are minimum order.

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ. Blockwise autoregressive representation and the static form

FHLZ: The model in static form

Start again with

$$A_n(L)\chi_{nt} = R_n\mathbf{v}_t.$$

Using $\mathbf{x}_{nt} = \chi_{nt} + \xi_{nt}$,

$$A_n(L)(\mathbf{x}_{nt} - \xi_{nt}) = R_n\mathbf{v}_t,$$

that is:

$$A_n(L)\mathbf{x}_{nt} = \mathbf{z}_{nt} = R_n\mathbf{v}_t + A_n(L)\xi_{nt},$$

$$\mathbf{z}_{nt} = R_n\mathbf{v}_t + \Xi_{nt}.$$

Under mild assumptions we can prove that Ξ_{nt} is idiosyncratic. Thus the original dynamic model has been transformed into a **static** one. We use the spectral density of the χ's to obtain the matrices $A_n(L)$, thus $\mathbf{z}_{nt}$. Then use the static model to estimate $\mathbf{v}_t$.

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ. Blockwise autoregressive representation and the static form

FHLZ: The forecasting equation

Inverting the autoregressive equation

$$A^k(L)\chi_t^k = R^k v_t$$

we get

$$\chi_t^k = [A^k(L)]^{-1} R^k v_t = B_0^k v_t + B_1^k v_{t-1} + \cdots$$

This is used to predict χ_{t+h}^k:

$$\chi_{t+h|t}^k = B_h^k v_t + B_{h+1}^k v_{t-1} + \cdots$$

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ. Blockwise autoregressive representation and the static form

FHLZ: Reordering the panel

Under our assumptions, the particular grouping into $(q + 1)$-dimensional subvectors of χ_{nt} does not matter in population. **However, in practice the grouping makes a difference.** We deal with this problem by **randomizing** the groupings and **averaging** over the results.

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ. Blockwise autoregressive representation and the static form

FHLZ. Forecasting equation

SW: $x_{1,t+h|t} = \boldsymbol{\alpha}(L)\mathbf{F}_t + \beta(L)x_{1t}, \quad x_{1,t+h|t} = \boldsymbol{\alpha}\mathbf{F}_t$

FHLR: $x_{1,t+h|t} = \boldsymbol{\alpha}(L)\mathbf{G}_t + \beta(L)x_{1t}, \quad x_{1,t+h|t} = \boldsymbol{\alpha}\mathbf{G}_t$

FHLZ: We use $\hat{A}^k(L)$ and $\hat{u}_t$:

$$\boldsymbol{\chi}_{t+h|t}^1 = \hat{A}^k(L)^{-1}\hat{R}^k E[\hat{v}_{t+h}] = \hat{B}_h\hat{v}_t + \hat{B}^{h+1}\hat{v}_{t-1} + \cdots$$

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

DATA

The dataset consists of 115 U.S. macroeconomic and financial time series observed at monthly frequency between January 1959 and September 2014. See the paper for details and the transformations necessary to achieve stationarity. We concentrate on inflation $(1 - L^{12})\mathrm{CPI}_t$ and industrial production IP_t.

We use the subsample up to December 1984 to calibrate the forecasting equations, the remaining sample for the comparison. Estimation and pseudo out-of-sample forecasting are based on a ten-years rolling window.

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Parameters. Calibration

SW: (i) Number of static factors,
     (ii) Include lagged factors in the forecasting equation or not,
     (iii) Include lagged x in the forecasting equation or not.

FHLR: (i), (ii), (iii), plus
     (a) Number of dynamic factors,
     (b) Window and kernel in the spectral estimation.

FHLZ: (a) and (b), plus
     (α) Degree of the polynomials $A^k(L)$,
     (β) Number of permutations of the panel.

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Average results

	IP				CPI			
h	FHLZ	FHLR	SW	AR	FHLZ	FHLR	SW	AR
1	0.949	0.910	0.926	1.000	0.958	1.066	1.085	1.000
3	1.048	0.944	0.902	1.000	0.891	1.001	1.066	1.000
6	0.981	0.893	1.020	1.000	0.887	1.013	1.146	1.000
12	0.906	0.840	0.987	1.000	0.972	1.120	0.985	1.000

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

Testing the relative forecasting performance

We use the test by Giacomini and Rossi to test the Null of equal performance at each point in time of two forecast methods, whose prediction errors are denoted by $\hat{\epsilon}_{t+h}$ and $\hat{\eta}_{t+h}$:
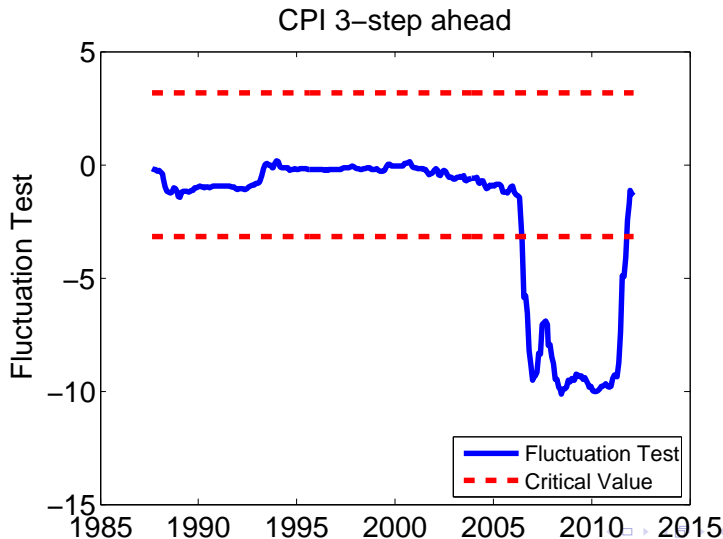
$$F_{t,m} = \frac{1}{\hat{\sigma} m^{1/2}} \left(\sum_{j=t-m/2}^{t+m/2} \hat{\epsilon}_{j+h}^2 - \sum_{j=t-m/2}^{t+m/2} \hat{\eta}_{j+h}^2 \right)$$

where between brackets we have the difference between squared prediction errors smoothed over a rolling window of size $m + 1$. This is normalized using m and $\hat{\sigma}$ is a Heterosketasticity and Autocorrelation Consistent estimator of the variance of $\epsilon^2 - \eta^2$. G. and R. compute critical values that, if crossed, signal that one method outperforms the other.
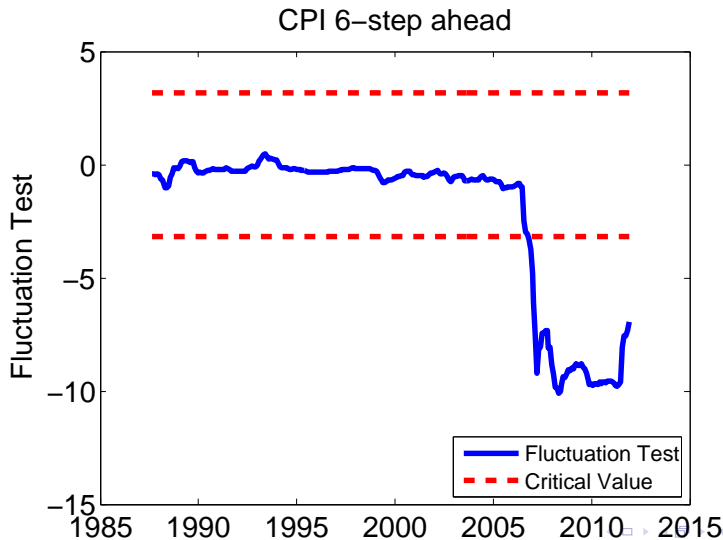
Introduction
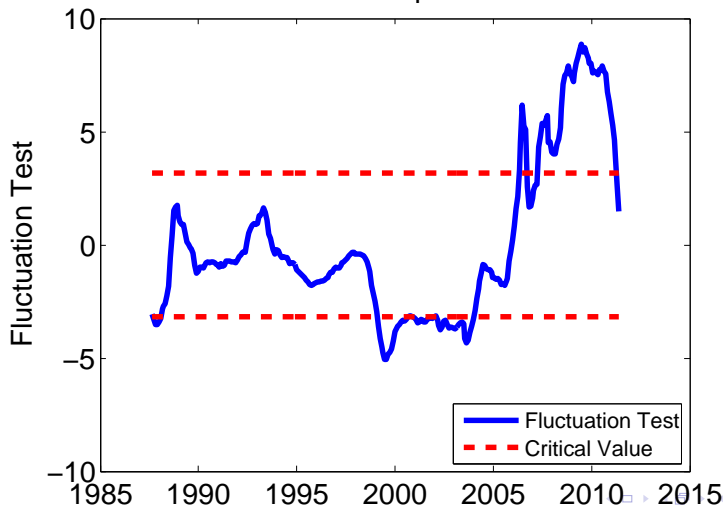Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ vs SW

CPI 1−step ahead

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ vs SW

CPI 3–step ahead

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ vs SW

CPI 6–step ahead

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLZ vs SW

CPI 12−step ahead

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLR vs SW

IP 1−step ahead

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLR vs SW

IP 3−step ahead

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLR vs SW

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results

FHLR vs SW

IP 12–step ahead

Introduction
Prediction. Finite-dimensional factor space
Prediction.Infinite-dimensional factor space (frequency domain)
Empirical results
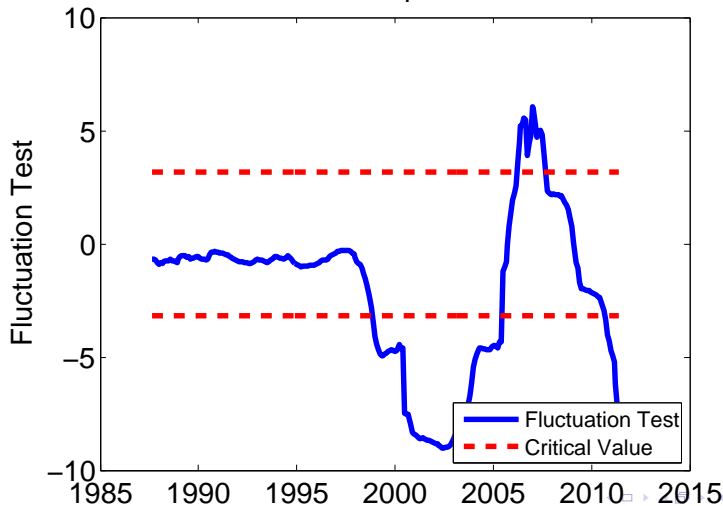
Conclusion

Good results of frequency-domain methods as compared to time domain. In particular, FHLZ very good with nominal variables; FHLR very good with real variables.
Potential for improving on FHLZ: we obtain many predictors by reordering the variables. We averaged over them and also made some attempts with LASSO etc., with some good results. Still to be explored systematically.