Combined Density Nowcasting in an Uncertain Economic Environment

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Motivation - Data Uncertainty

- Economic forecasts and decision making in real time are made in a changing data and changing/incomplete model environment
  - Data are released with a substantial time delay.
  - Data have mixed frequencies.
  - Many series are subsequently revised.

- Factor models provide a convenient and efficient tool to exploit information in a large panel of time series in a systematic way by summarizing the information of the many data releases within a few common factors.
  - Forecasting: Stock and Watson (2002a,b), Forni et al. (2005) and Boivin and Ng (2005)
  - Nowcasting: Giannone, Reichlin and Small (2008), Banbura et. al (2012), Banbura and Modugno (2014).
  - But most studies focus on one factor model and point forecast...

Motivation - Model Uncertainty and Incompleteness

- There is considerable uncertainty regarding model specification, e.g., choice of variables to include in the data set, choice of number of factor; possible incomplete model set
  - Forecast combination (Bates and Granger (1969), Timmermann (2006) and Clark and McCracken (2009,2010))
    - Combining factor models: Koop and Potter (2004), Kuzin et. al (2013)
- If decision maker's loss function is not quadratic or world is non-linear it no longer suffices to focus solely on first moments
  - Density nowcasting: Carriero et. al (2015), Marcellino et. al (2015), Aastveit et. al (2015)
- Combining predictive densities (Hall and Mitchell (2007), Jore et. al (2010))
  - Nowcasting: Aastveit et al. (2014)
  - Time varying weights: Koop and Korobilis (2012)
  - Time varying weights with learning and model set incompleteness: Billio et al. (2013) and Casarin et al. (2014).

Contribution of this paper - What we do

We introduce a Combined Density Nowcasting (CDN) approach applied to Dynamic Factor Models (DFMs)

- Combine predictive densities from a set of DFMs
  - Time-varying weights
  - Model set incompleteness
  - Combination weight uncertainty and learning

- The combined density is a convolution of a set of three probability density functions:
  1. Predictive density of the different models
  2. Weight density
  3. Combination scheme density.
  - Make use of Bayesian Sequential Monte Carlo methods to approximate 2. and 3, see Billio et al. (2013).

Contribution of this paper - Simulation results

- Simulation experiment to understand the role of data uncertainty and model set incompleteness at different data releases for nowcasting
- **Weak incompleteness**
  - Missing observations of the data, i.e. 'ragged edge' problem
  - The true model **is** part of our model set
- **Strong incompleteness**
  - Missing observations of the data, i.e. ragged edge problem
  - The true model **is not** part of our model set.
- When weak or strong incompleteness is present, our CDN approach outperforms BMA and all individual models

Contribution of this paper - Empirical results

- Nowcasting GDP growth, Signals of Model Incompleteness, and Probability of negative growth using U.S. real-time data.
  - Combine 4 different dynamic factor models and update the nowcast at 3 different points in time in each month of a quarter for the period 1990Q2-2010Q3.
  - For density nowcasting, CDN outperforms a selection strategy, Bayesian Model Averaging and the ex post best performing individual model.
  - The relative gains from using the CDN is largest in the early part of the quarter.
    - Uncertainty is highest.
    - Incompleteness plays a larger role.
  - CDN produces Time-Varying SD's of residuals signaling model incompleteness in recession periods and give probabilities of negative growth that provide good signals for calling recessions as a stage in business cycles in real time
    - Competitive with **S**urvey of **P**rofessional **F**orecasters probabilities of negative growth.

Individual Factor Models

Observation equation:

$$X_{t_m} = \chi_{t_m} + \xi_{t_m} = \Lambda F_{t_m} + \xi_{t_m} \tag{1}$$

where Λ is a $(T \times K)$ matrix of factor loadings, $F_m = \begin{pmatrix} f_{1t_m}, & \dots, & f_{Kt_m} \end{pmatrix}'$ is the static common factors and $\xi_{t_m} = \begin{pmatrix} \epsilon_{1t_m}, & \dots, & \xi_{nt_m} \end{pmatrix}'$ is an idiosyncratic component with zero expectation and $\Psi_{t_m} = E\left[\xi_{t_m}\xi'_{t_m}\right]$ as covariance matrix.

Transition equation (dynamics of the common factors follows a VAR):

$$F_{t_m} = AF_{t_m-1} + Bu_{t_m} \tag{2}$$

where $u_{t_m} \sim WN(0, I_s)$, B is a $(K \times s)$ matrix of full rank s, A is a $(K \times K)$ matrix where all roots of $\det(I_r - Az)$ lie outside the unit circle.

The idiosyncratic and VAR residuals are assumed to be independent:

$$\begin{bmatrix} \xi_{t_m} \\ u_{t_m} \end{bmatrix} \sim i.i.d.N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} R & 0 \\ 0 & Q \end{bmatrix} \right) \tag{3}$$

with R set to be diagonal.

Individual Factor Models

- Obtain predictions of quarterly GDP growth, y_{t_q}, by using a **bridge equation**.
    - The monthly factors F_{t_m} are first forecasted over the remainder of the quarter using equation (2).
    - To obtain quarterly aggregates of the monthly factors, $(F_{t_q} = F_{t_m}^{(3)})$
        - Prior to estimation: Transform each monthly variable to correspond to a quarterly quantity when observed at the end of the quarter(see Giannone, Reichlin and Small (2008)).
    - The nowcast of quarterly GDP growth (y_{t_q}), can then be expressed as a linear function of the expected common factors:

$$y_{t_q} = \alpha + \beta' F_{t_q} + e_{t_q} \tag{4}$$

- We obtain $p(F_{t_q+h}|I_K)$, $p(\widetilde{y}_{t_q+h}|F_{t_q+h}, I_K)$ and $p(\widetilde{y}_{t_q+h}|I_K)$ using the bootstrapping approach in Aastveit et al. (2014)

Constructing predictive densities

The bootstrap procedure in Aastveit et al (2014) is used to construct simulated forecasts: Let $\hat{A}_0 = [\hat{A}_1, \ldots, \hat{A}_p]$, $\hat{B}_0$, $\hat{u}_{0,t_m}$, $\hat{\xi}_{0,t_m}$, $\hat{\Lambda}_0$, $\hat{\alpha}_0$, $\hat{\beta}_0$, and $\hat{e}_{0,t_m+h_m}$ denote the initial point estimates. Then, for $d = 1, ..., 2000$:

1. Simulate $\tilde{F}_{t_m} = \sum_{i=1}^{p} \hat{A}_i \tilde{F}_{t_m-i} + \hat{B}_0 u^*_{t_m}$, where $u^*_{t_m}$ is re-sampled from $\hat{u}_{0,t_m}$.
2. Simulate $\tilde{X}_{t_m} = \hat{\Lambda}_0 \tilde{F}_{t_m} + \xi^*_{t_m}$, where $\xi^*_{t_m}$ is re-sampled from $\hat{\xi}_{0,t_m}$.
3. Based on $\tilde{X}_{t_m}$, re-estimate the model to get a new set of parameter and factor estimates. Use these to generate factor forecasts according to 2, where shock uncertainty is included by re-sampling from $\hat{u}_{0,t_m}$.
4. Estimate equation 4 based on the factor estimates in the previous step, and construct forecasts for $\tilde{y}_{T_m+h_m|T_m}$ where shock uncertainty is included by re-sampling from $\hat{e}_{0,t_m+h_m}$.

Combined Density Nowcasting with Dynamic Factor Models

Define I_K different Dynamic Factor model specifications including lagged data information. The density nowcast of GDP growth $p(y_{t_q+h}|I_K)$ is the convolution:

$$p(y_{t_q+h}|I_K) = \int_{\widetilde{Y}_{t_q+h}} \int_{W_{t_q+h}} p(y_{t_q+h}|\widetilde{y}_{t_q+h}, w_{t_q+h}, I_K) p(w_{t_q+h}|w_{t_q}) \\ dw_{t_q+h} p(\widetilde{y}_{t_q+h}|I_K) d\widetilde{y}_{t_q+h} \quad (5)$$

- $p(\widetilde{y}_{t_q+h}|I_K)$: is the predictive density of the K vector y_{t_q+h} following equation (4).
- $p(y_{t_q+h}|\widetilde{y}_{t_q+h}, w_{t_q+h}, I_K)$: is the combination scheme for the K different predictive densities with combination weights distributed as $p(w_{t_q+h}|w_{t_q})$.

Combined Density Nowcasting with Dynamic Factor Models

Gaussian combination that allows for model incompleteness:

$$p(y_{t+h}|\widetilde{y}_{t+h}, w_{t+h}, I_K) \propto \exp\{-\frac{1}{2\sigma^2}\left(y_{t+h} - \widetilde{y}_{t+h}^{'} w_{t+h}\right)^2\} \tag{6}$$

where w_{t+h} is a vector containing the K values for the combination weights and $\widetilde{y}_{t+h}$ contains the K predicted values from a distribution with density $p(\widetilde{y}_{t+h}|I_K)$.

The combination disturbances, defined as ζ_{t_q+h}, are estimated and their distribution provide a probabilistic measure of the incompleteness of the model set. The model in equation (6) is:

$$y_{t_q+h} = \tilde{y}_{t_q+h}' w_{t_q+h} + \zeta_{t_q+h} \tag{7}$$

with $\zeta_{t_q+h} \sim \mathcal{N}(0, \sigma^2)$.

Combined Density Nowcasting with Dynamic Factor Models

Combination Weights have a probabilistic distribution in the unit interval and they are nonlinear/logistic transforms for all K models, given as

$$w_{k,t_q+h} = \frac{\exp\{z_{k,t_q+h}\}}{\sum_{j=1}^{K} \exp\{z_{j,t_q+h}\}},\ k = 1, ..., K \tag{8}$$

Dynamics of Weights

$$p(z_{t_q+h}|z_{t_q}, \tilde{y}_{t_q-\tau:t_q}) \propto \exp\left\{-\frac{1}{2}\left(\Delta z_{t_q+h} - \Delta e_{t_q+h}\right)' \Lambda^{-1} \left(\Delta z_{t_q+h} - \Delta e_{t_q+h}\right)\right\} \tag{9}$$

$z_{t_q+h} = z_{t_q+h-1} - \Delta e_{t_q+h} + \textit{disturbance}$ is a latent process evolving over time which describes the contribution of each model in the nowcasting performance combination

Learning Function based on past predictive performance

$$e_{k,t_q+h} = (1-\lambda) \sum_{i=\tau}^{t_q} \lambda^{i-1} e_{k,i}^2,\ k = 1, .., K$$

where λ is a discount factor, and $(t_q - \tau + 1)$ is the length of the learning period. Different scoring rules can be applied depending, see Gneiting (2011).

Non-linear Filtering Combination Algorithm and Parallelization

- The **convolution** equation (5) involves a multiple integral. We use sequential Monte Carlo integration to solve it with respect to weights and combination scheme. We use also draws from the K individual predictive densities.
- The conditional density $p(y_{t_q+h}|I_K)$, given past observations, can be approximated as follows.

  • First, draw M independent values $\tilde{y}^j_{t_q+h}$ with $j = 1, ..., M$ from the conditional predictive density of the K models, $p(\tilde{y}_{t_q+h}|I_K)$

  • Conditionally on $\tilde{y}^j_{t_q+h}$ obtain the particle sets $\Xi^{i,j}_{1:t_q+h} = \{\mathbf{z}^{i,j}_{1:t_q+h}, \omega^{i,j}_{t_q+h}\}^N_{i=1}$, with $j = 1, \ldots, M$, $\mathbf{z}_t = (w_{t_q+h}, \theta)$.

  • Simulate $y^{i,j}_{t_q+h}$ from $p(y_{t_q+h}|\mathbf{z}^{i,j}_{t_q+h}, \tilde{\mathbf{y}}^j_{t_q+h})$, $i = 1, ..., N$, $j = 1, ..., M$, and obtain

$$p_{N,M}(\mathbf{y}_{t_q+h}|\mathbf{y}_{1:t_q+h-1}) = \frac{1}{MN}\sum_{j=1}^{M}\sum_{i=1}^{N}\omega^{i,j}_t \delta_{\mathbf{y}^{i,j}_{t_q+h}}(\mathbf{y}_{t_q+h})$$

Forecast evaluation

Root Mean Square Prediction Errors (RMSPE)

$$RMSPE_k = \sqrt{\frac{1}{t^*} \sum_{t=\underline{t}}^{\overline{t}} e_{k,t+h}}$$

where $t^* = \overline{t} - \underline{t} + h$, $\overline{t}$ and $\underline{t}$ denote the beginning and end of the evaluation period, and $e_{k,t+h}$ is the h-step ahead square prediction error of model k.

Logarithmic Score (LS)

$$LS_k = -\frac{1}{t^*} \sum_{t=\underline{t}}^{\overline{t}} \ln p(\tilde{y}_{k,t+h} | y_{1:t}), \tag{10}$$

for all k.

Simulation exercise

- We implement a simulation exercise to understand the usefulness of our CDN approach for nowcasting.
  - What is the role of incomplete data and model set incompleteness?
    - **Weak incompleteness**: forecaster produces nowcasts based on missing observations of data (ragged edge problem)
    - **Strong incompleteness**: if also DGP is **not** a part of the forecasters' model space.
- 4 simulation exercises
  - 60 quarters of recursive nowcasts
  - DGP1: DFM with 2 factors at the end of the sample.
  - DGP2: VAR(4) in GDP growth, unemployment rate, inflation and interest rate.

Simulation exercise

- The 4 simulation exercises.
  1. **Sim1**: Assume DGP1, estimate 4 individual DFMs with no missing data observations.
     - No incompleteness.
  2. **Sim2**: Assume DGP1, estimate individual DFMs with 1-4 factors, now with missing observations of data.
     - Weak incompleteness.
  3. **Sim3**: Assume DGP1, estimate individual DFMs with 1-4 factors based on only hard data (a subset of the 'true' data set), i.e. missing observations and misspecified models.
     - Strong incompleteness.
  4. **Sim4**: Assume DGP2, estimate individual DFMs with 1-4 factors based, i.e. missing observations of data and misspecified models.
     - Strong incompleteness.

Simulation results

	BMA	Best model	CDN
	Sim1: No incompleteness		
LS	-0.251	**0.224**	0.074
MSPE	0.028	0.025	**0.024**
	Sim2: Weak incompleteness		
LS	-3.882	-3.875	**-0.459**
MSPE	0.198	0.161	**0.147**
	Sim3: Strong incompleteness		
LS	-4.359	-4.328	**-0.457**
MSPE	0.241	0.240	**0.169**
	Sim4: Strong incompleteness		
LS	-0.567	-0.555	**-0.325**
MSPE	0.205	0.186	**0.112**

The table shows results for the 4 simulation exercises.

Empirical exercise

- We produce density nowcasts/backcasts for GDP growth at 11 different points in time
  - We use real-time data for 120 monthly leading indicators for the U.S. economy
  - We combine 4 different DFMs (a model with 1 to 4 factors)
  - Evaluation period is 1990q2-2010q3.
  - We use the 2nd release of GDP as "actual" when evaluating forecast accuracy

We consider three different model specification strategies:

1. BMA: A Bayesian model averaging approach based on predictive likelihood.
2. SEL: A selection strategy where we recursively pick the model with the highest realized cumulative log score at each point in time throughout the evaluation period.
3. CDN: Our Combined Density Nowcasting approach applied to DFMs

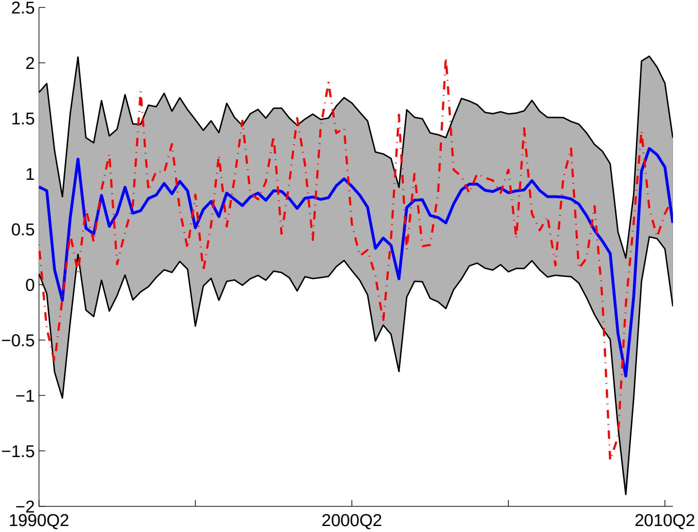
- In addition we include the ex post best model (Ex Post).

Data and blocks

Block	Time	Horizon
	Nowcasting	
1	Start of first month of quarter	2-step ahead
2	10th of first month of quarter (after inflation release)	2-step ahead
3	Around 20-25th of first month of quarter (after GDP relase)	1-step ahead
4	Start of second month of quarter	1-step ahead
5	10th of second month of quarter (after inflation release)	1-step ahead
6	Around 20-25th of Second month of quarter	1-step ahead
7	Start of thirds month of quarter	1-step ahead
8	10th of Third month of quarter (after inflation release)	1-step ahead
9	Around 20-25th of third month of quarter	1-step ahead
	Backcasting	
10	Start of fourth month of quarter	1-step ahead
11	10th of fourth month of quarter (after inflation release)	1-step ahead

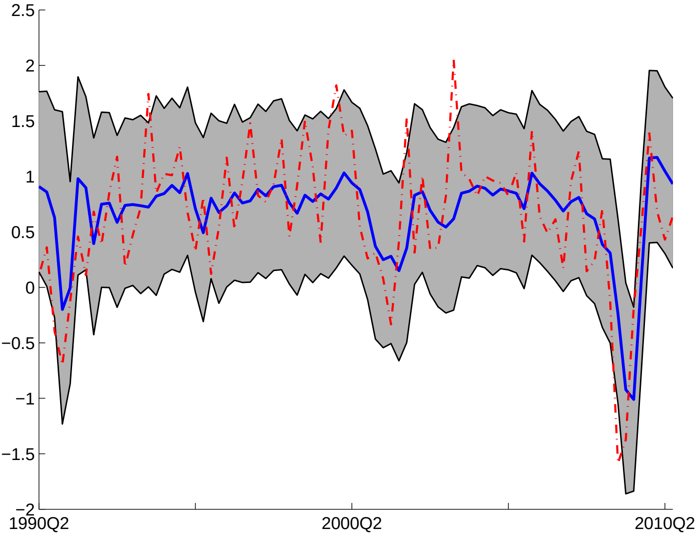
The table shows time in the quarter and forecast horizon for the 11 blocks.

Combined Density Nowcasts, block 1



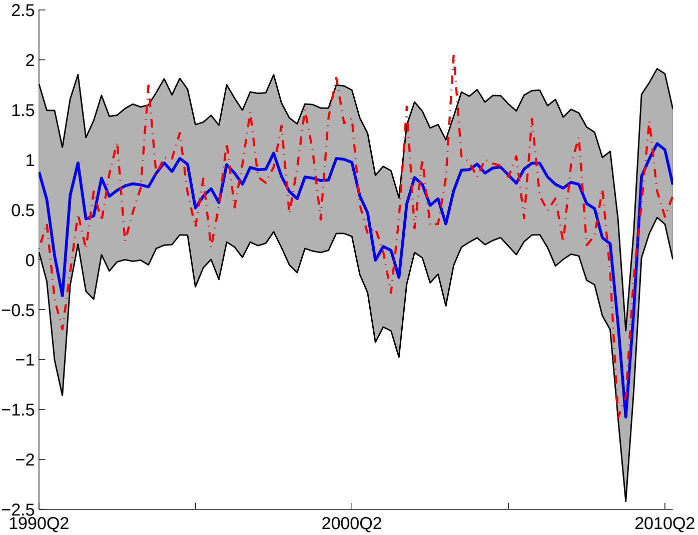
90% credibility intervals.

Combined Density Nowcasts, block 5



90% credibility intervals.

Combined Density Nowcasts, block 11

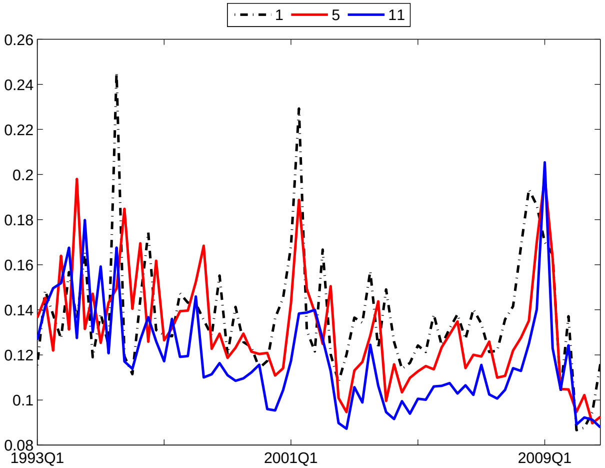


90% credibility intervals.

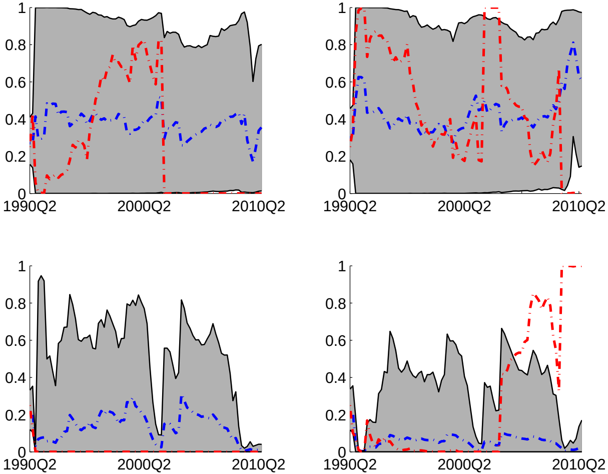
Point and density nowcasts of GDP growth

	BMA	SEL	Ex Post	CDN
		Block 1		
LS	-1.441	1.124	0.926	**0.590**
MSPE	0.583	0.988	**0.524**	0.542
		Block 2		
LS	-1.101	1.117	0.954	**0.715**
MSPE	0.317	1.032	0.959	**0.924**
		Block 3		
LS	-0.980	0.987	0.977	**0.814**
MSPE	0.289	0.989	**0.983**	1.025
		Block 4		
LS	-0.892	0.997	0.978	**0.862**
MSPE	0.275	0.991	**0.977**	1.007
		Block 5		
LS	-0.768	0.991	0.961	**0.897**
MSPE	0.241	0.990	**0.969**	1.002
⋮	⋮	⋮	⋮	⋮
		Block 11		
LS	-0.610	0.995	0.952	**0.931**
MSPE	0.187	0.991	**0.974**	0.989

Standard deviation of incompleteness

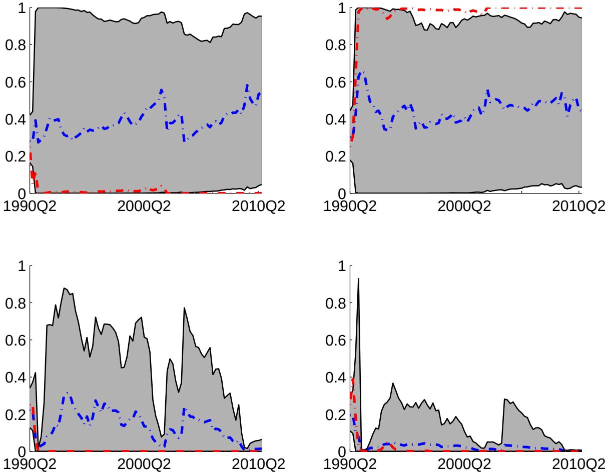


Standard deviation of the combination residuals for incomplete model sets.



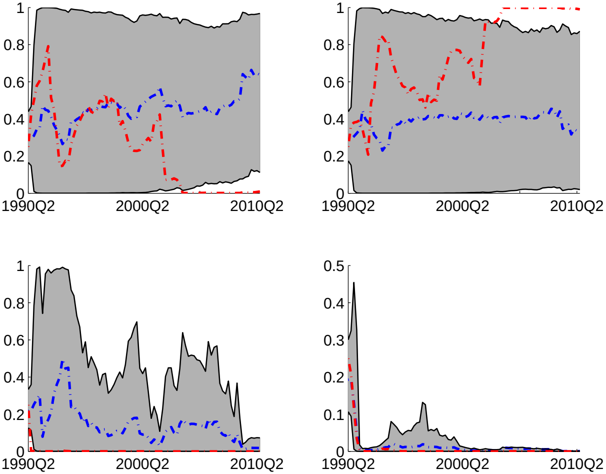
90% credibility intervals of the model posterior weights and their mean (dotted blue lines), BMA weights (dotted red lines).

Time-varying weights with learning, block 5



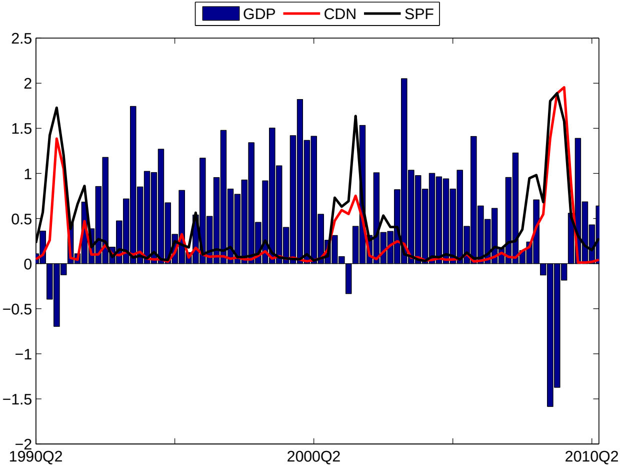
90% credibility intervals of the model posterior weights and their mean (dotted blue lines), BMA weights (dotted red lines).

Time-varying weights with learning, block 11



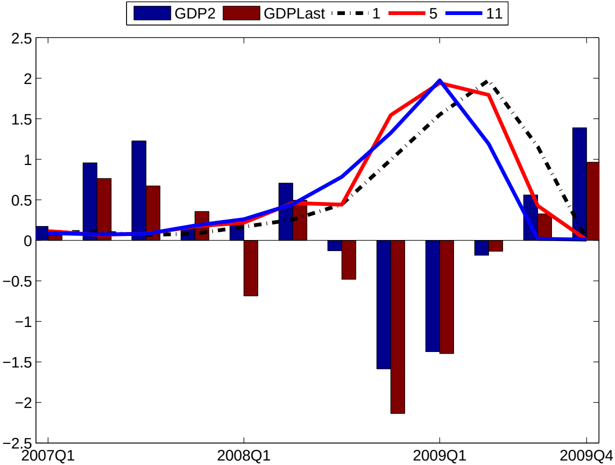
90% credibility intervals of the model posterior weights and their mean (dotted blue lines), BMA weights (dotted red lines).

Probabilities of negative growth, block 5



Probabilities over time of negative quarterly growth given by the CDN approach and SPF. The red and black lines plot the probabilities scaled by 2 (therefore covering the interval [0,2]); the bars plot the realization.

Probabilities of negative growth, Great Recession



Probabilities of negative quarterly growth during the Great Recession period provided by the CDN approach at different blocks during the quarter. The black dotted line, and the red and black solid lines plot the probabilities scaled by two (therefore covering the interval [0,2]) from the CDN approach at Block 1, Block 5 and Block 11, respectively. The blue and red bars plot the realizations measured as the second available estimate of GDP and the last available estimate of GDP (November 2014 vintage).

Summary

- We introduce a Combined Density Nowcasting (CDN) approach, applied to dynamic factor models, that accounts for time varying model uncertainty in order to provide more accurate nowcasts of predictive densities.
- We demonstrate in a simulation exercise the role of incompleteness for nowcasting.
- In an empirical exercise we use U.S. real-time data and show that;
  - For density nowcasting, CDN approach outperforms a selecting strategy, Bayesian Model Averaging and the ex post best performing individual model.
  - The relative gains from using the CDN approach is largest in the early part of the quarter.
    - Uncertainty is highest.
    - Incompleteness plays a larger role.
  - The CDN approach produce probabilities of negative growth that provides a good signal for calling recessions in real time
    - Competitive with SPF probabilities of negative growth.