Localized Fully Modified OLS Estimation of Cointegrating Relationships in an Integrated Locally Stationary Framework

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Abstract

Since the 1980ies, cointegration analysis has developed into one of the major modelling concepts in time series econometrics, with applications in a wide range of fields even outside economics and finance. In its simplest form the concept refers to the prevalence of linear combinations of integrated processes – or I(1) processes – that are stationary processes – or I(0) processes. The underlying assumption of cointegration analysis is one of stationary "innovation processes", such as in typical applications: stationary growth rates, stationary (log) price changes or stationary deviations from equilibrium. It is, however, not undisputed whether the data are generated by some underlying stationary mechanism. Contexts in which this assumption has been questioned include the great moderation or, of course, non-constant volatilities on financial markets. Thus, it may be of interest to extend cointegration analysis to account for changing "stationarity patterns" over time.

Consequently, as a natural extension, we develop cointegration analysis in a framework where the observed processes are modelled as integrated *locally stationary processes* (see, e.g., Priestley, 1965; Dahlhaus, 1997) rather than as integrated (stationary) processes. In particular we focus on a "cointegrating regression" setup, i.e., we consider:

$$y_{t,T} = x_{t,T}'\beta + u_{t,T} \quad (1)$$

$$x_{t,T} = x_{t-1,T} + v_{t,T}, \quad (2)$$

where the joint vector process $\eta_{t,T} = [u_{t,T}, v_{t,T}']'$ is a *locally stationary process* as considered by, e.g., Dahlhaus (2012) in the following form:

$$\eta_{t,T} = \sum_{j=0}^{\infty} \phi_j(\frac{t}{T})\varepsilon_t, \tag{3}$$

with $\sup_j ||\phi_j(\frac{t}{T})|| \geq \frac{k_1}{j^{1+k_2}}$ for all $j \leq j_0$, some $k_1 > 0$ and $k_2 > 1$. Additionally the coefficient functions $\phi_j(u),\ u \in [0,1]$, are assumed to be sufficiently smooth and to fulfill some additional technical conditions, e.g., to exclude "cointegration" amongst the regressors $x_{t,T}$, which is a typical (identifying) assumption also in standard cointegrating regression analysis. The process ε_t is assumed to be i.i.d. standard normally distributed.[1] Under the stated assumptions, the second order properties, or equivalently the *local spectral density* functions, are smoothly varying over u. Note that due to our linear regression setup we keep the "cointegrating relationship", i.e., the linear relationship between the series that leads to a locally stationary process ($[1, -\beta']'$), time-invariant and only allow the underlying process $\eta_{t,T}$ to have time-varying local stationarity properties. This setup reflects the fact that in many applications a particular focus lies on cointegrating relationships of a very particular structure, e.g., in purchasing power parity analyses one is interested whether the log real exchange rate is stationary (or now locally stationary), not whether an arbitrary and potentially time-varying linear combination of the nominal exchange rate and the prices in the two considered countries is stationary (or now locally stationary). Clearly, for other questions time-varying relationships may well be of interest.

As is common in the cointegration literature, we allow for both regressor endogeneity and error serial correlation in (1) and (2), now both of a time-varying nature. For this setting we derive the following results: First, the OLS estimator of β is shown to be consistent with a limiting distribution that is a function of Brownian motions with time-varying variances and where additive bias terms occur. The nuisance parameter dependency of this limiting distribution renders inference based on the OLS estimator difficult, a feature similarly present in standard cointegration analysis. Note that in order to derive OLS consistency, we first establish some underlying functional central limit results for the partial sum processes, which may be of independent interest. Second, based on the OLS limit an estimator inspired by the fully modified OLS (FM-OLS) estimator of Phillips and Hansen (1990) is developed. This estimator, labelled *localized* FM-OLS estimator, features similar transformations as the original FM-OLS estimator: (i) the dependent variable is *locally*

[1] The normality assumption can be weakened to an i.i.d. assumption together with an assumption on sufficiently many finite moments at the expense of more cumbersome proofs.

transformed to correct for the *local* dependence structure between the errors and the regressors and (ii) an additive bias term is subtracted. These corrections depend upon consistent estimators of the local long-run variances, which are also discussed in detail in the paper by extending the work of, e.g., Jansson (2002), from the stationary to the locally stationary framework. Finally, performing asymptotically standard inference requires – a potentially more surprising difference to the standard cointegration setting – something like "HAC-type" variance estimators related to the time-varying "heteroskedasticity" inherent in the locally stationary framework. Corresponding test statistics are constructed.

The theoretical analysis is complemented by a simulation study as well as an empirical application to the *forward rate unbiasedness hypothesis*.

JEL Classification: C12, C13, C32

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