Inference in VARs with Conditional Heteroskedasticity of Unknown Form

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Abstract

We derive a framework for asymptotically valid inference in stable vector autoregressive (VAR) models with conditional heteroskedasticity of unknown form. We prove a joint central limit theorem for the VAR slope parameter and innovation covariance parameter estimators and address bootstrap inference as well. Our results are important for correct inference on VAR statistics that depend both on the VAR slope and the variance parameters as e.g. in structural impulse response functions (IRFs). We also show that wild and pairwise bootstrap schemes fail in the presence of conditional heteroskedasticity if inference on (functions) of the unconditional variance parameters is of interest because they do not correctly replicate the relevant fourth moments' structure of the error terms. In contrast, the residual-based moving block bootstrap results in asymptotically valid inference. We illustrate the practical implications of our theoretical results by providing simulation evidence on the finite sample properties of different inference methods for IRFs. Our results point out that estimation uncertainty may increase dramatically in the presence of conditional heteroskedasticity. Moreover, most inference methods are likely to understate the true estimation uncertainty substantially in finite samples.

JEL classification: C30, C32

Keywords: VAR, Conditional heteroskedasticity, Residual-based moving block bootstrap, Pairwise bootstrap, Wild bootstrap

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1 Additional Information

Error terms in many econometric models for financial and macroeconomic time series are often uncorrelated but not identically and independently distributed (i.i.d). A popular example of deviations from independence include the case of conditional heteroskedasticity as e.g. in daily financial time series of asset returns but also macroeconomic time series as the monthly growth rates in industrial production, money, exchange rates, interest or inflation rates. Conditional heteroskedasticity and other departures from the i.i.d. assumption patterns have been documented in many empirical examples in the literature, see for instance Gonçalves & Kilian (2004). Moreover, these time series are often analyzed within vector autoregressive (VAR) models. VAR models are a popular econometric tool to summarize the dynamic interaction between the variables included in the VAR system. Many applications in applied macroeconomics and finance (see e.g. Sims (1992), Bernanke & Blinder (1992), Christiano, Eichenbaum & Evans (1999), Kim & Roubini (2000), Brüggemann, Härdle, Mungo & Trenkler (2008), Alter & Schüler (2012)) use VARs and conclusions are based on statistics obtained from the estimated VAR model. These statistics include e.g. Wald tests for Granger-causality, impulse response functions (IRFs) and forecast error variance decompositions (FEVDs). Inference on these statistics is typically based either on first order asymptotic approximations or on bootstrap methods. The deviation from i.i.d. errors as e.g. the presence of conditional heteroskedasticity invalidates a number of standard inference procedures for the quantities of interest, such that the application of these methods may lead to conclusions that are not in line with the true underlying dynamics. Therefore, in many VAR applications there is a need for inference methods that are valid if errors are uncorrelated but not independent.

In the time series context the existing literature makes some suggestions for valid inference under conditional heteroskedasticity. For instance, Gonçalves & Kilian (2004, 2007) consider inference on autoregressive (AR) parameters in univariate autoregressions with conditional heteroskedasticity. They show using a martingale difference sequence (mds) assumption on the errors that wild and pairwise bootstrap approaches are asymptotically valid and may be used to set up t-tests and confidence intervals for individual parameters. In addition, they also document that in finite samples the bootstrap methods are typically more accurate than the usual first-order asymptotic approximations based on robust standard errors. Also using an mds assumption, Hafner & Herwartz (2009) focus on Wald tests for Granger-causality within VAR models. They use both heteroskedasticity-consistent asymptotic inference as well as wild bootstrap methods, and find that especially the bootstrap methods provide more reliable inference.

Although the presence of conditional heteroskedasticity in time series data has been exploited in the VAR context for structural identification of shocks (see e.g. Rigobon (2003), Normandin & Phaneuf (2004) and Herwartz & Lütkepohl (2014)), the implications for inference e.g. on structural impulse responses have not been analyzed in detail yet. Popular statistics of interest in this context include responses to orthogonalized shocks, forecast error variance decompositions and tests for instantaneous causality, see e.g. Lütkepohl (2005, Chapter 2)). Inference on these statistics is more complicated as it requires to consider the joint asymptotic behavior of estimators for both VAR slope parameters and the parameters of the VAR innovation covariance matrix. The joint distribution is well explored in the case of i.i.d. innovations, see e.g. Lütkepohl (2005, Chapter 3). Although joint asymptotic inference in case of uncorrelated but non-independent errors is discussed in the framework of weak vector autoregressive-moving average (VARMA) models (see Boubacar Maïnassara & Francq (2011)), the implications of a departure from i.i.d. innovations for inference on structural impulse responses in VAR models is not well understood in the econometric literature.

To fill this gap, we analyze how relaxing the i.i.d. assumption in stable VAR models affects the limiting properties of estimators of both the VAR slope parameters and the unconditional innovation covariance matrix. In the following we refer to the vector autogressive slope parameter matrices simply as the 'VAR parameters', while the unconditional innovation covariance matrix is referred to as 'variance parameters'. In this paper, we contribute to the literature in a number of directions. First, we allow for quite general deviations from i.i.d. errors by using a mixing assumption on the error terms. Thus the results obtained in the paper cannot only be used in case of heteroskedastistic innovations but also cover more general innovation processes. The mixing assumptions enables us to make use of a fairly general result for weak VARMA derived in Boubacar Maïnassara & Francq (2011). We give a corresponding joint central limit theorem for the VAR case and show that our result is a special case of the one given in Boubacar Maïnassara & Francq (2011). Moreover, we also show how an explicit and simplified expression for the corresponding covariance matrix may be obtained for the VAR case. Second, we show how using an additional mds assumption further simplifies the structure of the joint covariance matrix. Thereby, we complement Hafner & Herwartz (2009) by providing a complete proof for the asymptotic results in the VAR case. In contrast to an i.i.d. error term set-up which leads to a block-diagonal asymptotic covariance matrix, see Lütkepohl (2005, Chapter 3), it turns out that the estimators of the mean and variance parameters are asymptotically correlated in general. A result corresponding to ours has been found by Ling & McAleer (2003) and Francq & Zakoïan (2004) for (vector) autoregressive moving average ((V)ARMA) models with generalized autoregressive conditional heteroscedastic (GARCH) innovations in terms of the estimators of the (V)ARMA and GARCH parameters.

We also analyze the theoretical properties of different bootstrap approaches commonly used in the VAR context. Our first result indicates that neither the wild bootstrap (recursive-design and fixed-design) nor a pairwise bootstrap work under mixing conditions only, as they cannot mimic the proper limiting distribution. Adding an additional mds error assumption is sufficient to ensure that bootstrap inference on the VAR parameters is consistent. Interestingly, we find that the recursive- and fixed-design wild bootstrap as well as the pairwise bootstrap that have been considered by Gonçalves & Kilian (2004, 2007) and Hafner & Herwartz (2009) turn out to lead to asymptotically invalid inference on (functions of) the innovation covariance matrix in the presence of conditional heteroskedasticity. The same holds true for the blockwise wild bootstrap that was recently proposed by Shao (2011). These bootstrap approaches fail in replicating the asymptotic variance of the innovation covariance estimator, which is a function of the fourth moments' structure of the innovations. Moreover, the wild bootstrap turns out to be inappropriate even in case of i.i.d. errors in case of inference on (functions of) the innovation covariance matrix.

As an alternative to the asymptotically invalid bootstrap methods mentioned above, we suggest to use a residual-based moving block bootstrap. The idea of the block bootstrap has been proposed by Künsch (1989) and Liu & Singh (1992) to extend the seminal bootstrap idea of Efron (1979) to dependent data. This and related approaches that resample blocks of time series data have been studied extensively in the literature, see e.g. Lahiri (2003) for an overview. In this paper, we prove that the residual-based moving block bootstrap (MBB) results in asymptotically valid joint inference on the VAR and variance parameters if suitable mixing assumptions are imposed. Since the block length in the MBB is assumed to grow to infinity with the sample size (at an appropriate rate), the MBB is capable of capturing the higher moment structure of the innovation process asymptotically. Therefore, the MBB is indeed able to correctly replicate the limiting covariance matrix of the innovation covariance estimator.

We illustrate the importance and implications of the theoretical results by studying inference on IRFs that are functions of both the VAR parameters and the innovation covariance parameters. This type of IRFs are of major importance in typical applied VAR studies. We provide simulation evidence on the finite-sample properties of corresponding first-order asymptotic approximations and of various bootstrap approaches. We draw two main lessons from our simulation study. First, applied researchers have to be aware that estimation uncertainty may dramatically increase if non-i.i.d. innovations are present. Second, in many situations the true sampling variation of the IRF estimators is understated by most of the inference procedures. This, in turn, may lead to (bootstrap) confidence intervals for impulse response coefficients being too narrow at short horizons. Accordingly, applied researchers should interpret their results with caution.

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