A Prediction Perspective of Unstable Processes

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Abstract

Prediction has long been a vibrant topic in modern probability and statistics. In addition to the traditional applications of prediction in finding optimal forecast and model selection, it is argued in this paper that prediction principle can also be used to unify critical phenomena, in particular, stationary and unstable time series. Although the notion of nearly unstable models has become one of the important concepts in time series econometrics, its role from a prediction perspective is less developed. Based on moment bounds for the extreme-value (EV) and least squares (LS) estimates, asymptotic expressions for the mean squared prediction errors (MSPE) of the EV and LS predictors for a nearly unstable first-order autoregressive (AR(1)) model are derived in this paper. These asymptotic expressions offer useful insight to the finite sample approximations of the MSPE for time series with near unit-root, from which a class of nearly unstable models can be developed. These nearly unstable models not only offer a smooth transition from the stationary regime to the unit-root case, they also shed light to the fundamental question of linking the MSPE between the stationary and unstable models.

KEY WORDS: Extreme-value predictor, least squares predictor, mean squared prediction error, nearly unstable, positive error.