The impact of network connectivity on factor exposures, asset pricing and portfolio diversification

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The need for understanding the propagation mechanisms behind the recent financial crises lead the increased interest for works associated with asset interconnections. In this framework, network-based methods have been used to infer from data the linkages between institutions. In turn, those connections have implications for the evaluation of systemic risk. The literature is still debating on the definition of systemic risk and how it differentiates from or overlaps with systematic risks. In this paper, we elaborate on this and make a step forward by introducing network linkages into linear factor models, thus allowing for the interdependence between asset connections and systematic risks. Networks are used to infer the exogenous and contemporaneous links across assets, and impacts on several dimensions. From a factor exposure perspective, network links act as inflating factor for systematic exposure to common factors, and allow for cross-asset exposures to factors due to the presence of the network. In turn, the presence of networks and factors, has potential implications for pricing. Nevertheless, we show that those implications have a role only at the local, or short-term, level, while over the long-run their effect is negligible. Furthermore, the power of diversification is reduced by the presence of network connections, and we analytically show that network links reduce the diversification potential but at the same time could allow for absorption of risks. Finally, our modeling framework is coherent with empirical evidences associated with standard linear factor model. By fitting a (misspecified) linear factor model under our data generating process (allowing for the presence of network links), the model provides residuals are correlated and heteroskedastic, and the factor exposures become time-varying. We support our claims with an extensive simulation experiment.

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1 Introduction

The term "Systematic risk" is a well established concept that derives from the seminal work on portfolio choice proposed by Markowitz (1952) and extended in a general equilibrium framework by Sharpe (1964), Lintner (1965a,b), and Mossin (1966) and in the Arbitrage Price Theory model by Ross (1976). It refers to the risk an investor of a well-diversified portfolio is exposed to, which stems from the dependence of returns to common factors.

On the other side, the definition of "Systemic risk" is not well defined throughout the literature and, as a result, can be measured from a wide range of perspectives. According to Acharya and Yorulmazer (2002), Nier et al. (2007) and De Bandt et al. (2000) systemic risk materialises through (1) "pure" contagion, (2) feedback effects from endogenous fire sales, (3) herding behaviour causing informational contagion, and (4) exposure to common factors. Hartmann (2002) argues systemic risk stems from either build-up imbalances, contagion or large shocks.

The broad definition provided above links contagion risk to systemic risk as well as exposure to common factors, that in principle is largely related to systematic risk. A natural statistical model for capturing systemic risk exposure due to linkages between institutions is a network model, which is commonly used to describe features of a network of connections.

In this paper we provide a unique framework for systematic risk and network connections and estimate the feedbacks among network exposures and common factors and the impact of them on the risk exposures and risk premia of stock returns. More specifically, we look to the the interactions of the four ways through which a broad definition of systemic risk materialize, i.e. the relationships between (i) "pure" contagion, (ii) feedback effects from endogenous fire sales that could be well captured by a network model, (iii) herding behaviour causing informational contagion and (iv) exposures to common factors that could be considered *per se* as systematic risk exposure.

A growing literature investigates the role of interconnections between different firms and sectors, functioning as a potential propagation mechanism of idiosyncratic shocks throughout the economy. Acemoglou et al. (2011) use network structure to show the possibility that aggregate fluctuations may originate from microeconomic shocks to firms; Billio, Gray, Getmansky, Lo, Merton and Pelizzon (2014) use contingent claim analysis and network measures to highlight interconnections among sovereign, banks and insurances. There are several other contribution in the literature on network analysis: see Billio, Getmansky, Lo, and Pelizzon (2012), Diebold and Yilmaz (2015) and Hautsch, Schaumburg, and Schienle (2012, 2013) and Barigozzi and Brownlees (2014). Network interconnections and the effects called network externalities that arises from small and local shocks that can become big and global is a possibility discarded in standard asset pricing and macro-economics models due to a "diversification argument". As argued by Lucas (1977), among others, microeconomic shocks would average out and thus, would only have negligible aggregate effects. Similarly, these shocks would have little impact on asset prices. However, there is also a growing literature on the role of sectorial shocks in macro fluctuations. Examples include Horvath (1998, 2000), Dupor (1999), Shea (2002), and Acemoglu et al. (2012). Morevoer, Ang et al. (2006), among others, show that idiosyncratic volatility risk is priced in the cross-section of expected stock returns, a regularity that is not subsumed by size, book-to-market, momentum, or liquidity effects. From a theoretical point of view Wagner (2010), Ozsoylev and Walden (2011), Allen et al. (2012), Buraschi and Porchia (2013) and Branger et al. (2014) arrive at similar conclusions. Ahern (2013) empirically documents a positive market price of centrality, i.e., more central assets earn higher expected returns.

The need for understanding the propagation mechanisms behind the recent financial crises leads to an increased interest for works associated with systemic risks. In this framework, network-based methods described above will be used to infer from data the linkages between institutions (or companies). Part of the literature postulates that systemic risk is strictly related (if not equal) to systematic risk and therefore there is no need to distinguish among the two. With this paper instead we argue that it is important to disentangle the channels through which risk propagates.

In fact, the contribution of this paper to this literature is to propose a modelling framework where network interconnections and common factors risks co-exist. The proposed model is a variation of the traditional CAPM/APT model where networks are used to infer the exogenous and contemporaneous links across assets. We also provide a number of generalizations for our approach to make it more flexible coherent with the empirical evidences, for instance allowing for asset-specific reaction to network links and introducing time-variation in networks.

By building on the newly introduced model, we provide a number of theoretical elements and empirical evidences based on a simulation framework. At first, focusing on the returns dynamic and the common factor exposure we show that the presence of asset interconnection acts as an inflating factor to the exposure on common risk sources. Moreover, we are able to disentangle the exposure to common factors that is structural, that is present even in the case of no network connections, from the exposure associated with network links. Similar argument applies to the shocks impacting on an asset return, where network relations expose assets to other asset's shocks. From a risk perspective, our approach allows us to decompose the risk of a single assets (or a portfolio) into four components: the two classical systematic and idiosyncratic components and (i) the impact of the asset interconnections on the systematic risk component, that is the contribution of network exposure to the systematic risk component and (ii) the effect of interconnections on the idiosyncratic risk on the systematic risk component, that is the amplification of idiosyncratic risks that generates systematic/non diversifiable risk. Building on this result, we show how diversification benefits are reduced in the presence of network connections. Moreover, by combining the return dynamic with the variance decomposition, we can verify that our model is consistent with the presence of correlation and heteroskedasticity among traditional linear factor model residuals, thus providing a rational for empirical evidences found in the literature.

Finally, we also evaluate the impact of networks on the estimation of risk premiums and show that the premiums estimated by our approach and by a traditional linear factor model are equivalent in the long-run (under some assumption on the evolution of the network over time). However, our approach allows for local (conditional) expected returns that change according to changes in the network structure, and thus leading to price changes even if the risk premiums are time-invariant.

The remainder of the paper is organized as follows. Section 2 describes network models. Section 3 presents our model combining network links and factor exposure, while Section 4 introduces a set of generalizations making the model more flexibly. Section 6 presents the simulation analysis and Section 7 concludes.

2 Network Models in Finance

Network models have seen an extremely diverse array of applications: in the social sciences with studies related to social networking on websites such as Facebook, in the natural sciences with application to protein interactions, in government intelligence where they are used to analyse terrorist networks, in politics with application to bill co-authorship, in economics with potential used in labour markets analysis, and many other areas. In finance, network models have most frequently been used to assess financial stability. In fact, interconnections among financial institutions create potential channels for contagion and amplification of shocks to the financial system that can be also propagated to the "real economy".

Applications in this area have gauged considerable interest in the aftermath of the 2007-2009 financial crisis. Network representation of interconnections ranges from linkages extracted from balance-sheet information to connections estimated by means of econometric approaches from either market data, accounting data or macroeconomic data.

The majority of such "real-world" networks have been shown to display structural properties that are neither those of a random graph, nor those of regular lattices.

In order to evaluate the relevance and the price of interconnections in the financial system it is fundamental to understand all of the channels by which small and local shocks can become big and global.

Empirical network modelling has been conducted for assessing asset pricing linkages via contagion (Allen and Gale 2000; Dasgupta 2004; Leitner 2005, Billio, Getmansky, Lo, and Pelizzon (2012), Diebold and Yilmaz (2014) and Hautsch, Schaumburg, and Schienle (2012, 2013), Brownlees (2014)), linkages via balance sheets (Cifuentes et al 2005; Laguno and Schreft 2001), and how failures of institutions result from mutual claims on each other (Furfine 2003; Upper and Worms 2004; Wells 2004). Babus and Allen (2009) provide a review of network models in finance.

Much of the empirical finance literature has focused on "direct"contagion arising from firms' contractual obligations. Direct contagion occurs if one firm's default on its contractual obligations triggers distress (such as insolvency) at a counterparty firm. Researchers' simulations using actual interbank loan data suggest that "domino defaults "arising from contractual violations are very unlikely, (see Furfine (2003) Eisinger et al. (2006), Upper and Worms (2004); Mistrulli (2011); Degryse and Nguyen (2007), Van Lelyveld and Liedorp (2006) and Alves et al (2013)) though they can be highly destructive in the event that they do materialise.

Contractual obligations are not the only means by which small and local shocks can spread and generate perverse externalities. Focusing only on direct contagion underestimates the risk of financial crisis given that other important channels exist like common exposures, fire sales, illiquidity spirals and, information spillover. For example, in its survey Upper (2011) reports that simulations using actual interbank loan data suggest that domino defaults are very rare events, and Abbassi, Brownlees, Hans and Podlich (2014) shows that model network structures for a sample of German banks based on CDS data are only marginally explained by direct connections through interbank exposures and common exposures to similar asset classes extracted by accounting data.

The approach that we follow in this paper is that both direct and indirect interconnections extracted from accounting or direct exposures data and market data could co-exist and have implication on the dynamic of the returns of financial assets. Therefore, our approach is very general. We first concentrate on interconnections that could be estimated from market data and then we provide a theoretical extension of the model where also direct linkages like balance-sheet exposures or common exposures to similar asset classes could be included in the framework.

The advantage of using market data to extract linkages has relevant advantages: the data are easily available, have higher frequency (that is more information, and a more up-to-date view of links) e and the linkages extracted from market data are forward looking in contrast to balance-sheet/accounting data that provide a pictures of the actual exposures (and might be seen thus as backward-looking). The forward looking interpretation can also supported by the general idea that market prices can be seen as reflecting information available to traders/operators/market participants, and, in equilibrium, correspond to the discounted value of future dividends (thus with a link to fundamental valuations of stocks).

Formally, we could represent networks as nodes that are connected (in general) to a subset of the network total number of nodes, where connections represent links across nodes. A financial system could be represented as a network structure where nodes represent assets or the value of financial or non financial institutions, and shocks on one asset/institutions are transmitted to the connected ones.

Networks are, in general, graphically represented, and we also provide some examples in the empirical section. Nevertheless, networks have an equivalent (square) matrix representation. Let us call W the $K-$dimensional square matrix representing a network composed by K financial assets/companies. Each entry $w_{i,j}$ represents the possible connection between assets i and j. A zero entry indicates that the two assets are not connected, while a non-null entry indicates the existence of a connection. Depending on the approach adopted to estimate the network, non-null entries might differ one from the other, that is they track the strength/intensity of the connection, or might be simply equal one to the other, and thus just indicate the existence of a connection. An example of the last case is the following matrix:

$$W = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \tag{1}$$

where note that the diagonal contains only null elements (each asset is not influencing itself) and the network is not symmetric as the first asset is connected to the fourth one, but the opposite is not true.

In general, networks also convey a further element, the direction of the link. If links are all bidirectional, the network is symmetric. By convention, in the present paper we assume that a non null element $w_{i,j}$ indicates the existence of a link between assets i and j with an effect from j to i.

Interestingly, matrices similar to that of equation (1) are very common in other economic and statistic applications, those concerning research and studies associated with spatial econometrics and spatial statistics. In these fields, subjects (like towns, buildings, regions) are neighbour one to the other in a physical way, and the W matrices represent the neighbouring relations with entries possibly associated with the physical distance existing between two subjects; they are normally called *spatial matrices*, and are commonly row-normalized.

Matrix representation of financial networks might thus be seen as the financial parallel of spacial matrices. Clearly, neighbouring relations are no more physical, but are the outcome of a specific model, measuring or estimation approach. Going back to the graphical representation of networks, where nodes are connected one to the other, we might state that connected nodes (assets/firms) are thus neighbour.

Finally, we stress that, if we consider matrices monitoring only the existence of the connection across assets, we adhere to the concept of “first order contiguity ”where a unit entry denotes the existence of a connection and the fact that two assets are neighbour, see LeSage (1999). In addition, by convention in spatial statistic/econometrics, the main diagonal of the W matrix contains zero elements.

In the following, we will clarify how network connections, as monitored by the matrix W will convey relevant information on the evolution of asset returns. In doing that, we do not restrict ourselves to a specific structure of W, that is with a W monitoring the existence of a connection and/or the intensity of the link, but will propose a model which can be used with any form of W. Moreover, according to Elhorst (2003), we will normalize W, so that, if we are monitoring only the existence of the connection, we equalize the impact of each unit on all other units. We will further discuss the normalization of W in the a following section.

Later, when moving to the empirical part, we will also briefly discuss alternative methods that can be followed to estimate the existence of a connection across two assets.

3 The systematic effects of network exposure

Since the seminal works of Sharpe (1964), Lintner (1965a,b), and Mossin (1966) linear returns models have attracted a huge interest in the financial economics literature, and have had an extraordinary impact on both research and practice. In the last decades, multifactor generalizations of the CAPM model have been proposed and are now as diffused as the single factor model. The first multifactor models stem from the work of Ross (1976) on the arbitrage pricing theory, and the most commonly used approaches in pricing take now into account the developments of Fama and French (1993 and 1995), and Carhart (1997), leading to the so-called three-factor and four-factor CAPM models, respectively.

Our starting point is a multifactor model, nesting all the previous cases, which we take as a general case where network exposure can be introduced. We thuc consider a linear specification where a K-dimensional set of time t risk asset returns, which we denote by

R_t, depends on a set of M *observable* zero-mean risk factors:

$$R_t = \alpha + \beta F_t + \varepsilon_t. \tag{2}$$

In equation (2) α is a $K-$dimensional vector of intercepts, β is a $K \times M$ matrix of parameters monitoring the exposure of the risky assets to the common factors included in the $M-$dimensional vector F_t; finally, and ε_t is the vector of idiosyncratic shocks. Note that, the notation we use, and thus also the following generalizations, can be applied to any collection of risk factors. However, for reasons explained below, the risk factors should not be recovered by means of *statistical* approaches, such as principal component analysis or the estimation of a latent factor model, but must be observed variables.

If we take a pricing perspective, and assume that the market is in equilibrium, then, the model intercept can be replaced by the vector of expected returns

$$R_t = \mathbb{E}\left[R_t\right] + \beta F_t + \varepsilon_t. \tag{3}$$

Moreover, expected returns depend on the factor risk premiums Λ obtaining

$$\mathbb{E}\left[R_t\right] = r_f + \beta\Lambda. \tag{4}$$

The four-factor CAPM allows decomposing the total risk of the assets into the sum of two components:[1]

$$\mathbb{V}\left[R_t\right] = \beta\Sigma_F\beta' + \Omega_\varepsilon, \tag{5}$$

where $\mathbb{V}\left[\cdot\right]$ is the variance operator, $\mathbb{V}\left[F_t\right] = \Sigma_F$ is the covariance matrix of the common factors, and $\mathbb{V}\left[\varepsilon_t\right] = \Omega$ is the covariance matrix of the idiosyncratic shocks. The first term on the right represents the systematic contribution to the total risk, while the second term is the idiosyncratic risk contribution. The same decomposition of the total

[1]This holds for any multifactor model.

assets risk applies also to a generic portfolio formed with the K assets. If we take a vector of portfolio weight ω,[2] the portfolio returns satisfy the following equalities

$$\begin{aligned} r_{p,t} &= \omega' R_t \\ &= \omega' \mathbb{E}\left[R_t\right] + \omega' \beta F_t + \omega' \varepsilon_t \\ &= \mathbb{E}\left[r_{p,t}\right] + \beta_p F_t + \varsigma_t, \end{aligned} \tag{6}$$

where $\mathbb{E}\left[r_{p,t}\right] = r_f + \beta_p \Lambda$. Moreover, we know that the total risk of the portfolio is given as

$$\begin{aligned} \mathbb{V}\left[r_{p,t}\right] &= \omega' \beta \Sigma_F \beta' \omega + \omega' \Omega_\varepsilon \omega \\ &= \beta_p \Sigma_F \beta_p' + \sigma_\varsigma^2 \end{aligned} \tag{7}$$

This framework has relevant implications on portfolio risk and diversification. If we take a diversification point of view, the final purpose is to control or sterilize the impact of asset idiosyncratic risks on the total portfolio risk. This corresponds to the willingness of achieving the following limiting condition

$$lim_{K \to \infty} \omega' \Omega_\varepsilon \omega = \tilde{\sigma}^2 > 0 \tag{8}$$

where $\tilde{\sigma}^2$ is a small quantity depending on the idiosyncratic shock variances and correlations, as well as on the portfolio composition. In a simplified setting, assuming that idiosyncratic shocks are uncorrelated, that their variances are set to an average value $\bar{\sigma}^2$ and taking an equally weighted portfolio, we have the following well-know result

$$lim_{K \to \infty} \omega' \Omega_\varepsilon \omega = \frac{1}{K} \bar{\sigma}^2 = 0, \tag{9}$$

showing that diversification allows sterilizing the idiosyncratic shocks.

[2] We assume that portfolio weights sum at 1 but we do not exclude short selling.

In this framework the focus is on the shocks impact, since we know that the systematic risk component cannot be diversified out, as it is driven by common factors. Therefore, in the multifactor model, the introduction of new assets allows a contraction of the contribution of the idiosyncratic component to the total risk of the portfolio, but has, in average, no effects on the systematic components.[3]

Our paper aims at introducing in an asset pricing multifactor model the impact of contemporaneous links that exist across assets, when those are captured by a network. As discussed in the previous section, networks provide information on the existence of links and might also convey details on the intensity of the link existing across assets. Therefore, we aim at coupling the systematic and idiosyncratic risks with a sort of network risk that would introduce in the model the assets cross-dependence beyond that captured by common factors. Given this further element we will then evaluate the effects on traditional uses of the multifactor model.

Let us assume that the risky assets are interconnected and that those links can be represented by a network. The network relations, as observed in the previous section, can be, in some sense, forward looking or represent the actual state of the connections across assets. From this point onward, we will assume that, indifferently from the approach adopted for the estimation of the network, the network will impact on the contemporaneous relations across assets. Starting from this assumption, we have to partially reconsider the interpretation of a general multifactor model. In fact, if we postulate the existence of contemporaneous relations across risky assets, we must acknowledge that those are not explicitly accounted for in 2. Moreover, the common factors capture the dependence of each risky asset from common sources of risk, but the presence of interconnections implies that risky assets are exposed to the movements (both systematic and idiosyncratic) of other risky assets. We might label this additional component as network exposure. In addition, risky assets might differ in terms of interconnections with other assets, and can thus be affected by an additional form of heterogeneity going beyond those associated

[3]Nevertheless, we note that, by means of short selling and when a risk free asset is present, we might be able to build portfolios that annihilate the effect of at least some risk factors.

with the different exposure to common factors and with the relevance of the own idiosyncratic risk. As a consequence, the beta matrix with respect to common factors that can be recovered from 2 cannot be directly linked to both the interconnections and to the source of *network* heterogeneity across risky assets.

One possible way of indirectly recovering the network exposure is to interpret the model in 2 as a reduced form model where reduced form parameters (the betas and the error covariance) are functions of structural parameters. The latter thus include the *true* exposure to common factors, the exposure to other assets due to the interconnections (or network exposure) and the *structural* idiosyncratic shock's variance.

To shed some light on the previous points we rewrite the model in 3 as a structural simultaneous equation system

$$A\left(R_t - \mathbb{E}\left[R_t\right]\right) = \bar{\beta} F_t + \eta_t \tag{10}$$

where the matrix A captures the contemporaneous relations across assets and it co-exist with the common factors which are here considered as exogenous variables. In 10 the covariance of η_t represents the structural idiosyncratic risk while the parameter matrix A is associated with assets interconnections, and thus with a network. Further details on the last aspect will be given in few paragraphs. If we translate the model 10 into a reduced form, we have

$$R_t = \mathbb{E}\left[R_t\right] + A^{-1}\bar{\beta} F_t + A^{-1}\eta_t \tag{11}$$

where we stress two relevant elements. Firstly, we observe that the reduced form parameters of the linear factor model, which can be consistently estimated by least squares methods, are non-linear functions of the interconnections across assets (the matrix A) and of the structural exposure to common factors (the matrix $\bar{\beta}$). Secondly, the covariance matrix in 2 is also influenced by the presence of asset's interconnections. Note that, if we postulate that i) a network structure exists, and thus assets are interconnected, ii) that there are just four common factors, and then iii) we estimate the linear factor model in 2 without taking into account the network, we have by construction that the shocks are correlated.[4] Therefore, the empirical evidences of idiosyncratic shock correlation found on the residuals of a four-factor CAPM model might be due to the exclusion of contemporaneous relations as shown by the results of Ang, Hodrick, Xing, and Zhang (2006): idiosyncratic volatility risk is priced in the cross-section of expected stock returns, a regularity that is not subsumed by size, book-to-market, momentum, or liquidity effects. In addition, if we assume that the network links affect the matrix A, and estimate model (11), the residuals covariance will be a function of the network links. Thus, if network links are not known, they might be estimated by looking at the covariance of $A^{-1}\eta_t$, as in Barigozzi and Brownlees (2014). However, in such a case the economic interpretation of network links might be difficult to recover and potentially exposed to estimation error.

We also highlight a further aspect. If the common factors are estimated by means of statistical approaches rather than being observed variables, the network exposure, if present, will be totally destroyed. In fact, *statistical* factors are generally estimated from a reduced form model. Therefore, if we neglect the network exposure and adopt, say, principal component analysis, or fit a latent factor model, it might happen that one of the identified factors represent a sort of proxy of or a biased estimate of the network exposure, with possible further biases on the estimated factor loadings. Such a problem might be overcome by estimating a latent factor model accounting for contemporaneous links across assets.

Our approach aims are re-introducing contemporaneous relations into the multifactor model thus allowing to recover both the impact of network exposure as well as the exposure to common factors. Note that both elements co-exist, and network exposure can be seen as an additional *common* risk source going beyond that of common factors. We might even define the exposure to common factors as the exogenous systematic risk exposure, while the network exposure can be labelled as an endogenous systematic risk exposure.

[4] This holds if we assume that A is not diagonal. However, this is an inconsequential restriction as if A is diagonal we do not have contemporaneous relations across assets.

Notably, in this way, the idiosyncratic risks will be defined as *structural* and, at least in principle, should be less correlated than the shocks in 2.

The simultaneous equation system in 10 poses serious challenges for the estimation of the matrix A. We overcome this potential problem by resorting to network links. If we postulate the existence of network connections, exogenously provided by direct exposures and indirect exposures across risky assets, we can state that linked assets are *neighbors*. Such a wording is very common in the spatial econometrics literature where relations across entities (towns, areas, countries) depend in many cases on the physical (geographical) distances and are collected in a proximity matrix that identifies neighbor entities. In a financial framework, the network links can be easily recast in a proximity matrix W as mentioned in Section 2. The proximity matrix can be used to impose a structure on the matrix A. Given the matrix W, as extracted from a network, we can easily specify a spatial autoregressive (SAR) model (see Anselin, 1988, and LeSage and Pace, 2009):[5]

$$R_t - \mathbb{E}\left[R_t\right] = \rho W \left(R_t - \mathbb{E}\left[R_t\right]\right) + \bar{\beta} F_t + \eta_t \tag{12}$$

where the (scalar) coefficient ρ captures the response of each asset to the returns of other assets, as weighted with the corresponding row of W. Moreover, we assume that the error term η_t has a diagonal covariance matrix, that is $\mathbb{V}\left[\eta_t\right] = \Omega_\eta$ is diagonal. Such an assumption is required for identification purposes as we will discuss in the model estimation section. If we assume, as we will do in the following, that the matrix W is known, the expected returns are conditional to the (known) W. To maintain a simplified notation we do not report the conditioning with respect to W in the returns expectations.

At the single asset level the model reads as follows

$$R_{i,t} = \mathbb{E}\left[R_{i,t}\right] + \rho \sum_{j=1}^{k} w_{i,j} \left(R_{j,t} - \mathbb{E}\left[R_{j,t}\right]\right) + \bar{\beta}_i F_t + \eta_{i,t} \tag{13}$$

[5] Anselin (1988) calls the model mixed-regressive spatial-autoregressive. We stick here to the simpler acronym adopted in LeSage and Pace (2009).

where $w_{i,i} = 0$, $w_{i,j} \geq 0$ and $\sum_{j=1}^{k} w_{i,j} = 1$. Taking a financial point of view, the coefficients in the vector $\bar{\beta}_i$ represent the exposure to the common factors, or *exogenous* exposure, while the coefficient ρ tracks the *endogenous* risk exposure which is influenced by the network structure, and thus called network exposure. Further insights on the interpretation of the model coefficients will be given in the following subsections.

The model in 12 can be rewritten in a more compact form as follows

$$(I - \rho W)(R_t - \mathbb{E}[R_t]) = \bar{\beta} F_t + \eta_t \tag{14}$$

thus highlighting the fact that spatial proximity and the associated SAR model give a structure to the contemporaneous relation matrix, which is now parametrized as

$$A = I - \rho W \tag{15}$$

The structural model now includes contemporaneous relations, driven by links or connections across asset, systematic components and asset specific shocks. We now elaborate on the relation between returns, risk, networks and risk factors.

3.1 Returns, networks and risk factors

The reaction of one asset to common factors and network exposure appears in a more clear way once we rewrite the model in a reduced form representation. In this way we highlight the impact of the network connections included in W on the reduced form parameters (the reduced form betas and the reduced form shock's covariance). The model reads as:

$$R_t = \mathbb{E}[R_t] + \mathcal{A}\bar{\beta} F_t + \mathcal{A}\eta_t \tag{16}$$

where $\mathcal{A} = A^{-1}$, $A = I - \rho W$ and we assume that A is non-singular. For simplicity, we focus on the case where the network exposure is driven by a single parameter, the ρ. However, all derivations and comments apply also to the more general parametrizations of the matrix A that we will introduce in Section 4.

From LeSage and Pace (2009) we take the following relation

$$(I - \rho W)^{-1} = I + \rho W + \rho^2 W^2 + \rho^3 W^3 \ldots, \tag{17}$$

where the term ρW monitors the effect of linked assets (in spatial econometrics, the neighbours), for instance if asset j is linked to asset i we have a non-null entry in W_{ij}. Differently, $\rho^2 W^2$ is associated with the effect on asset j induced by the assets linked to asset i (those called in spatial econometrics, the second-order neighbours). The latter relation can be further generalized to higher orders. Notably, the matrices W^j might also include a so-called *feedback loop* as, following the previous example, asset i can be linked to asset j (the relation is thus bi-directional), causing the matrix W^j to have non-null elements on the main diagonal. We stress that, despite the summation has infinite terms, by imposing that $|\rho| < 1$ we can easily ensure the effect of linked assets converges to zero. On the contrary, if $|\rho| > 1$ we might have explosive patterns. In general, the coefficient ρ takes values in the range $\left(\lambda_{min}^{-1}, \lambda_{max}^{-1}\right)$, with λ_{min} and λ_{max} are, respectively, the minimum and maximum eigenvalues of W. In the case of row-normalization of the W matrix, in spatial econometrics a commonly adopted range is $[0, 1)$. We will further elaborate on the values assumed by ρ in following sections.

By using 17 we can rearrange the model in 16 as

$$R_t = \mathbb{E}\left[R_t\right] + \bar{\beta} F_t + \sum_{j=1}^{\infty} \rho^j W^j \bar{\beta} F_t + \eta_t + \sum_{j=1}^{\infty} \rho^j W^j \eta_t. \tag{18}$$

Such a representation highlights that the impact of the common factors as well as of the idiosyncratic shocks on the risky asset returns can be decomposed into two parts. For both elements, the first component is the traditional, or direct, or structural impact, while the second component is the impact associated with the network exposure. We can thus define the following four elements:

a - $\bar{\beta}F_t$: the structural exposure to common factors;

b - $\sum_{j=1}^{\infty}\rho^j W^j \bar{\beta}F_t$: the network exposure to common factors;

c - η_t: the structural impact of idiosyncratic shocks;

d - $\sum_{j=1}^{\infty}\rho^j W^j \eta_t$: the network impact of idiosyncratic shocks.

Note that the network-related exposures depends on the structure of the matrix W as well as on the parameter monitoring the network impact, the ρ. A relevant remark comes from the network impact of common factors. Let's take for simplicity a specific common factor, that is, we focus on a single column of F_t and consider the impact of the $m-$th factor on the risky asset returns

$$\bar{\beta}_m + \sum_{j=1}^{\infty}\rho^j W^j \bar{\beta}_m. \tag{19}$$

Equation (19) provides two relevant insights.

At first, we note that the network exposure to common factors acts as a multiplier of the structural exposure if the ρ coefficient is positive (W elements are anyway positive). Therefore, shocks to the common factors will be amplified by: the presence of connections across assets, that is when, for asset i, the $i-$th row of W contains at least one non-null element; the change in the impact of network connections, that is when the ρ coefficient increases; by changes in the network structure, that is when the matrix W changes. Note that, if asset i is not connected to other assets, all products $\rho^j W^j \bar{\beta}_i$ are equal to zero. From a different viewpoint, the presence of network exposure allows us to decompose the betas into two components, a structural one, and a multiplier depending on the network structure, the W matrix. The estimation of a standard factor model where the data generating process includes network dependence across returns, will provide partial information, returning only the combination of the two components.

Now assume that for the risky asset i the $m-$th common factor is not relevant (that is $\bar{\beta}_{i,m} = 0$). In this case, in the standard linear factor models, the common factor will have no role in explaining the asset returns. However, when asset are linked and network exposures are taken into account, a common factor to which a risky asset has a zero structural exposure might still be relevant to explain the risky asset return evolution. Such an effect is not direct but induced from the network exposure and is associated with the existence of non null elements in the $i-$th row of the matrix W. Take for instance the following case

$$W = \begin{bmatrix} & \vdots & \\ \mathbf{0}_i & 1 & \mathbf{0}_{K-i-1} \\ & \vdots & \end{bmatrix} \tag{20}$$

where assets i is connected only to asset $i+1$ and subscripts denote the length of row vectors of zeros. Moreover, assume the following factor exposure for both assets

$$\bar{\beta} = \begin{bmatrix} & \vdots & & \\ \beta_{1,i} & 0 & 0 & 0 \\ \beta_{1,i+1} & \beta_{1,i+1} & 0 & 0 \\ & \vdots & & \end{bmatrix}, \tag{21}$$

where, in a multi-factor model, asset i is not exposed to factor 2 while asset $i+1$ is affected by the same risk factor, and both assets are exposed to factor 1. Asset i dependence on risk factors can thus be represented as

$$\beta_{M,i} R_t^M + \rho\beta_{1,i+1} F_{1,t} + \rho\beta_{2,i+1} F_{2,t} + \sum_{j=2}^{\infty} \left(\rho^j W^j \bar{\beta} F_t\right)|_i \tag{22}$$

where $|_i$ identifies the $i-$th element of a vector. Note that the last term on the right represents further elements that can be specified only through the knowledge of the entire W matrix. Therefore, even if a risky asset i is not (structurally) exposed to a common factor (in the previous example, factor 2), the common factor will play anyway role if it impacts on the returns of the assets to which i is linked.

Such a result can be further generalized by focusing, for instance, on sector specific risk factors. Those, in presence of a network exposure, despite being sector specific will have a systematic impact on all connected assets. Moreover, if we disregard the network exposure, we might also incur in the risk of misinterpreting the impact of risk factors. In fact, by estimating the reduced form model we might label as *common* a factor that in reality is structurally related just to a subset of the investment universe and impact on other assets only through network connections.

A similar property exists for the idiosyncratic shocks. In fact, if we assume they are uncorrelated, the existence of network connections implies that the structural shocks of one asset impacts on the returns of all the connected assets. Therefore, shocks on single assets can have effects on many other risky assets.[6]

From a pricing perspective, starting from the reduced from representation we can easily show that the expected returns equal

$$\mathbb{E}\left[R_t\right] = r_f + \bar{\beta}\Lambda + \sum_{j=1}^{\infty} \rho^j W^j \bar{\beta}\Lambda. \tag{23}$$

Expected returns are thus influenced by network links that amplify the compensation for being exposed to the common risk factors.Further, we note that the pricing result depends on, and is thus conditional to, the network structure, as summarized by W, which we assume to be known and time invariant. In fact, if we postulate that the ρ coefficient is positive and that the elements of W are all positive, the existence of links across assets induces higher expected returns as opposed to the case where links are absent. Moreover, bearing in mind the previous discussion, the expected returns might depend on risk premiums associated with factors to which a given asset is not directly

[6]Summary measures of the exposure to common factors and idiosyncratic shocks can be obtained by mimicking the approaches used in spatial econometrics. A discussion on this topic is included in LeSage and Pace (2009), see their section 2.7; these measures have been used in a financial framework by Asgharian et al. (2013). We also note that the decomposition of asset returns into four elements is equivalent to that of Abreu et al. (2005) for separating the standard impact of covariates from that due to the spatial links, and is thus an alternative to the impact measures of LeSage and Pace (2009).

(structurally) exposed.

In addition, we stress that the use of a network which is very dense, thus implying a W matrix almost full, will have further impacts. In fact, a full W implies that all idiosyncratic shocks are correlated. However, from our viewpoint, this correspond to an indirect evidence of model misspecification as an additional common factor is now present but not taken into account. As a consequence, such a common factor risk must be priced, and could generate the empirical evidences shown by Ang et al. (2006). The latter case could also correspond to an empirical evidence challenging the validity of the APT pricing approach. From a different viewpoint, our modeling framework still satisfy the assumptions required for APT. As we will show in the next section, the presence of a network exposure despite inducing correlation across the idiosyncratic shocks does not exclude the existence of diversification benefits. In turn, this is sufficient to guarantee the validity of the APT where risk premiums can be recovered from the reduced form model.

3.2 Risk decomposition

The model in 10 allows recovering a risk decomposition similar to that available for the standard linear factor models in 2. The starting point is the reduced form introduced at the beginning of the previous subsection, see 16. Equation 16 highlights that the estimation output of standard multifactor models can be coherent with the presence of contemporaneous links across assets. In fact, we can redefine $\beta = \mathcal{A}\bar{\beta}$ and $\varepsilon_t = \mathcal{A}\eta_t$, and estimate the reduced form mean parameters, the matrix β and the covariance of ε_t. However, this do not lead to the identification of the structural parameters: the structural factor loading $\bar{\beta}$, and the network related coefficient ρ included in $\mathcal{A}$. On the contrary, our purpose is to identify structural parameters of 10. Given the knowledge of structural parameters, the total variance of the risky assets can be written as follows

$$\mathbb{V}\left[R_t\right] = \mathcal{A}\bar{\beta}\bar{\beta}'\mathcal{A}'\sigma_m^2 + \mathcal{A}\Omega_\eta\mathcal{A}' \tag{24}$$

Despite being equivalent to the traditional risk decomposition of a multifactor model, 24 provides a relevant insight. In fact, both the systematic and idiosyncratic risk components are influenced by the presence of interconnections across assets as the matrix $\mathcal{A}$ appears on both the right hand site terms. This shows also that, if we estimate the reduced form model with standard linear methods, our evaluations of the systematic and idiosyncratic risk components are in reality a blend of the structural loadings and idiosyncratic risks with the network relations. Keiler and Eder (2013) suggest that the presence of spatial links could be interpreted as a systemic risk contribution. However, the previous decomposition provides an alternative view, where spatial dependence is not an additive source of risk but rather a multiplicative one, where the asset-specific effect cannot be easily recovered (as it depends on both the structure of the network and the associated W matrix and the spatial parameters in ρ).

Obviously, the same structure appears at the portfolio level where we have

$$\mathbb{V}\left[r_{p,t}\right] = \omega'\mathcal{A}\bar{\beta}\Sigma_F\bar{\beta}'\mathcal{A}'\omega + \omega'\mathcal{A}\Omega_\eta\mathcal{A}'\omega \tag{25}$$

Since our main focus is a portfolio of risky assets, we start elaborating on the last decomposition of the portfolio total risk. Nevertheless, we stress that comments similar to those later reported apply also to each risky asset return. We assume that we want to maintain a reference with the structural parameters $\bar{\beta}$ as they represent the impact of systematic movements on the portfolio. However, the existence of interconnections across assets is affecting such impact at the portfolio level, moving it away from that we would have observed if interconnections were not be present. The latter, common factor loading without interconnections, would equal $\omega'\beta$, but in reality, i.e. with interconnections, we have $\omega'\mathcal{A}\bar{\beta}$. We might thus interpret the product $\omega'\mathcal{A}$ as a transformation of portfolio weights, due to the impact of the interconnections across assets. The factor $\mathcal{A}$ amplifies or reduces the relevance of one asset compared to its true monetary weight in the portfolio.

Those modified weights represent the impact at the portfolio level of systematic shocks affecting the risky assets. The interconnections are thus matched with the portfolio weights rather than altering the betas. This is just a choice which we further motivate by the decomposition we now introduce.

We first note that, if assets interconnections are not present (that is when $\mathcal{A} = I$), the idiosyncratic risk equals Ω_η while the systematic risk component is $\bar{\beta}\Sigma_F\bar{\beta}'$. We rewrite portfolio variance decomposition in 25 by adding and subtracting the portfolio idiosyncratic and systematic variance components when those are not influenced by asset interconnections:

$$\mathbb{V}\left[r_{p,t}\right] = \omega'\mathcal{A}\bar{\beta}\Sigma_F\bar{\beta}'\mathcal{A}'\omega + \omega'\mathcal{A}\Omega_\eta\mathcal{A}'\omega \pm \omega'\bar{\beta}\Sigma_F\bar{\beta}'\omega \pm \omega'\Omega_\eta\omega \tag{26}$$

After rearranging, the total portfolio variance can be recast into a decomposition counting four different terms

$$\begin{aligned} \mathbb{V}\left[r_{p,t}\right] &= \underbrace{\omega'\bar{\beta}\Sigma_F\bar{\beta}'\omega}_{I} + \underbrace{\left(\omega'\mathcal{A}\bar{\beta}\Sigma_F\bar{\beta}'\mathcal{A}'\omega - \omega'\bar{\beta}\Sigma_F\bar{\beta}'\omega\right)}_{II} && (27)\\ &+ \underbrace{\omega'\Omega_\eta\omega}_{III} + \underbrace{\left(\omega'\mathcal{A}\Omega_\eta\mathcal{A}'\omega - \omega'\Omega_\eta\omega\right)}_{IV} && (28) \end{aligned}$$

We give the following interpretation to the four risk components:

I Is the structural systematic risk component that depends on the structural loadings from the common factors and from the covariance of the common factors; this is the *exogenous* systematic effect;

II Is the of asset interconnections on the systematic risk component, or first contribution of network exposure to the total risk; this is the *endogenous* systematic effect;

III Is the structural idiosyncratic component that depends only on the structural shocks covariance;

IV Is the effect of interconnections on the idiosyncratic risk, or second contribution of network exposure to the total risk; this might be interpreted as an *endogenous* amplification of idiosyncratic risks.

Note that by adding the second and fourth terms we obtain the total contribution of network exposure to the total portfolio risk. We finally notice that the model with assets interconnections gives the standard multifactor model if there are no interconnections, that is W is a null matrix, or, if the coefficient ρ is statistically not significant.

In addition, the network exposure impact on the idiosyncratic part of the variance implies that the diversification benefits might be endangered depending on the network structure. In fact, despite the fourth term will decrease with increasing cross-sectional dimension, the decrease speed will be smaller compared to the case without network effects.

Similarly to the standard linear factor model, we can recover analytical elements in a simplified setting. As we previously stated, the covariance matrix Ω_η is diagonal; we further assume that the diagonal elements are set to an average value $\bar{\sigma}^2 = 1$. In addition, we take an equally weighted portfolio and focus on the limiting case where all assets are connected (thus W has zeros only over the main diagonal, while off-diagonal terms equal $\frac{1}{K-1}$ after row-normalization). In this case, it can be shown that

$$\begin{aligned} \omega' \mathcal{A} \Omega_\eta \mathcal{A}' \omega &= \bar{\sigma}^2 \omega' \mathcal{A}\mathcal{A}' \omega \\ &= \frac{\bar{\sigma}^2}{K^2} \mathbf{i}_K' \mathcal{A}\mathcal{A}' \mathbf{i}_K \\ &= \frac{K+\rho^2}{(K+\rho)^2 (\rho-1)^2} \bar{\sigma}^2 \end{aligned} \tag{29}$$

where K is the asset number and $\mathbf{i}_K$ is a $K-$dimensional vector of ones.[7] Moreover, we have that

[7]In the special case considered the diagonal elements of $\mathcal{A}$ equals $\frac{(K-1)\rho - K}{\rho^2 + (K-1)\rho - K}$ and the off-diagonal

$$lim_{K\to\infty}\frac{K+\rho^2}{\left(K+\rho\right)^2\left(\rho-1\right)^2}\bar{\sigma}^2=0 \tag{30}$$

thus preserving the diversification benefit. However, the idiosyncratic risk contribution is higher than in the case without spatial dependence (i.e. with $\rho=0$). In fact, we can show that the above reported portfolio idiosyncratic risk is higher than $\frac{1}{K}\bar{\sigma}^2$ thus confirming that term IV is positive.

The previous model gives thus a framework where we can analyse the impact at the portfolio level of the interconnections we might observe across assets, and how those interconnections can endanger/limit the benefits of portfolio diversification. The following section provides some further evidences, examples and comments on a simulated framework.

4 Model generalizations

4.1 Heterogeneous network reaction

The model in (14) has, however, a very restricted structure. In fact, there is a single parameter, the ρ, driving the network exposure. This can be easily generalized by allowing for asset-specific responses to the network structure. We can thus modify the contemporaneous relation matrix of (15) into

$$A \;=\; I-\mathcal{R}W \tag{31}$$

where $\mathcal{R}=diag(\rho_1,\rho_2,\ldots,\rho_K)$ is a diagonal matrix. This model is similar to the fixed coefficient specifications for spatial panels discussed in Elhorst (2003). A clear advantage

elements are $\frac{-\rho}{\rho^2+(K-1)\rho-K}$. Moreover, the diagonal elements of $\mathcal{AA}$ equal $\frac{K\rho^2+[(K-1)\rho-K]^2}{[\rho^2+(K-1)\rho-K]^2}$ and the off-diagonal are $\frac{(K-1)\rho^2-2\rho[(K-1)\rho-K]}{[\rho^2+(K-1)\rho-K]^2}$. Summing up the elements in $\mathcal{AA}$ and simplifying we obtain the above reported result.

of such a structure is given by the possibility that assets have different network exposures, as for each asset the model becomes

$$R_{i,t} = \mathbb{E}\left[R_{i,t}\right] + \rho_i \sum_{j=1}^{k} w_{i,j} \left(R_{j,t} - \mathbb{E}\left[R_{j,t}\right]\right) + \bar{\beta}_i F_t + \eta_{i,t}. \tag{32}$$

To estimate the asset-specific parameters the network must satisfy an identification condition: each asset must be connected to at least one other asset. If this is not the case, the diagonal of matrix $\mathcal{R}$ must be restricted in such a way that not-connected assets will not have a network exposure. Further details will be discussed in the estimation section.

4.2 Time-change in the network structure

The spatial econometrics literature generally assumes that the spatial proximity matrix is time invariant. In fact, if the matrix W depends on physical measures, such as those is the space, those can be safely assumed constant over time. However, in a financial framework, the connections across assets might change over time for a number of reasons, some of them being, for instance, the occurrence of an unexpected market shock, mergers and acquisitions. Similar approaches have been adopted by Asgharian et al. (2013) and Keiler and Eder (2013). We mentioned in Section 2 that the network structure can be estimated on the basis of different approaches and data. The latter can be either time series and/or cross sectional data. Therefore, the networks might be estimated, with the same type of data, over different samples. Clearly, by changing the sample, we can easily obtain different networks, and the time-evolution of connections across assets is itself a relevant, but also expected, finding. Despite the time-variation of the networks, and still assuming the network exogenous with respect to the linear structural model,[8] the contemporaneous matrix can be further re-written as

[8]We might relax the exogeneity assumption by stating that the network are known conditionally to the past.

$$A_t = I - \mathcal{R}W_t \tag{33}$$

where we highlight that the network changes over time, and thus lead to a time-varying W matrix. In turn, this induce time-dependence on the A matrix, as well as on the reduced form parameter matrices, both on the betas as well as on the covariance of idiosyncratic shocks, that is, we have also heteroskedasticity. Nevertheless, we might postulate that the dynamic of W_t is smooth, and operates at lower time scales as compared to those monitoring the evolution of returns (for instance we can assume the W matrices change over years, or after specific events such as crises). Therefore, the heteroskedasticity is mild, and the betas are slowly evolving. The use of time-varying W matrices thus lead to a time change in the spatial dependence differing from the approach of Blasques et al. (2013) that obtain the same result by letting the $\mathcal{R}$ parameters being time-varying. We notice that, if the network exposure exist and the structural parameters in the matrix $\bar{\beta}$ are constant, the estimation of the reduced form model over different samples might suggest changes in the factor exposure. However, those changes are not present but due to the misspecification of the network relations. We remind that the expected returns are conditional to the W matrix. If the network exposure is time-varying, the expected returns, conditional to W_t will be time varying.

A further issue associated with the change over time of W_t is the normalization. In fact, if we let each single W_t to be row-normalized, we could reduce the impacts of changes in the network density: an increase in the number of assets linked to asset j would lead to a decrease of the impact coming from a single asset since the corresponding element of W_t would diminish. As a consequence, with the introduction of dynamic W_t we also suggest to consider a different normalization which we refer to as a *max row normalization.* Formally, an non normalized W_t^U will be normalized as

$$W_{i,j,t} = W^U_{i,j,t} \left(max_t \sum_{i=1}^{N} W^U_{i,j,t} \right)^{-1}. \tag{34}$$

We stress that, when conditioning on the network structure, the pricing equation assumes the following form (where we also introduced asset-specific coefficients for network exposure):

$$\mathbb{E}\left[R_t | W_t\right] = r_f + \bar{\beta}\Lambda + \sum_{j=1}^{\infty} \left(\mathcal{R}W_t\right)^j \bar{\beta}\Lambda. \tag{35}$$

The heterogeneity with respect to connections creates reactions to shocks on the common factors that differ across assets due to the different exposures of assets to the factors, but also due to the different impact of feedback loops coming from the underlying network structure. The change over time of the W_t matrix, or the presence of a structural break on the $\mathcal{R}$ coefficients (that we might locate in proximity of a crises or of an extreme event) creates abrupt changes in the expected returns with the consequence of relevant movements in stock prices. Therefore, the pricing, conditional to the network structure becomes a function of the network structure: if the network changes, the *local equilibrium* expected returns change. Alternatively, when we introduce a time-variation in the W matrices, or in the $\mathcal{R}$ elements, the APT still holds and with risk premiums estimated in the cross-sectional dimension starting from the reduced form model parameters and within a certain time interval. However, if we focus on a standard pricing model, we neglect such a potential local time-variation in expected returns. Note we refer to local equilibrium as the expected returns in (35) are conditional to the network structure. The long range equilibrium returns should be computed integrating with respect to the network dynamic.

4.3 Plurality of networks

This further generalization of the model refers to the possibility of constructing a network structure from different data, for instance cross-exposures or estimation of causality relations. This is both intuitive and feasible within our model. In fact, a-priori, we do not have information allowing us to order the alternative networks in terms of their relevance, nor we cannot exclude some of them. However, competing networks can be easily introduced in the model, allowing the data to provide guidance to network comparison. In fact, the contemporaneous relation matrix can be written as

$$A = I - \sum_{j=1}^{m} \rho_j W_j \tag{36}$$

where m different networks are jointly introduced into a model. The estimated parameters can then provide useful details on the relevance/preference of different network measures.[9] We also note that distance matrices W recovered from a network approach can be also jointly used with similar matrices obtained from different methods, such as on the basis of economic sector partitions of assets as in Arnold et al. (2013) and Caporin and Paruolo (2013), bilateral trades (Asgharian et al. 2013), or foreign direct investments (Fernandez-Avila et al. 2012).

4.4 Contributing to and receiving from networks

We further note that the use of a matrix $A = I - \mathcal{R}W$ lead to a focus on the impact of the network exposures where the asset-specific coefficients ρ_i represents the impact on i coming from the assets linked to i, or, from a different viewpoint, it is the loading of i from the network risk. We might, however, be interested on the effect of asset i on the other assets, having thus a ρ_i coefficient that represents the impact of i to the assets to which i is linked. We might see this as an outgoing effect of i to other assets through the network, or as a contribution of i to the network factor.

This can easily be achieved with a simple modification of the model, by replacing A with $B = I - W\mathcal{R}$. With such a change, the return equation (32) becomes

[9] We note that, when the network exposure parameter are asset-specific, the introduction of different W matrices requires some identification conditions that depend on the network structures.

$$R_{i,t} = \mathbb{E}\left[R_{i,t}\right] + \sum_{j=1}^{k} w_{i,j}\rho_j\left(R_{j,t} - \mathbb{E}\left[R_{j,t}\right]\right) + \bar{\beta}_i F_t + \eta_{i,t}. \tag{37}$$

We now note that the ρ_j coefficients represent the impact of the $j-th$ asset on the other assets. Moreover, if we consider the reduced form representation of the model, we have

$$R_t = \mathbb{E}\left[R_t\right] + \mathcal{B}\bar{\beta}F_t + \mathcal{B}\eta_t \tag{38}$$

where $\mathcal{B} = B^{-1}$. The reduced form betas can again we seen as a by-product of both the structural risk exposure, the $\bar{\beta}$ and the inflating factor coming from the network, the $\mathcal{B}$. However, the structure of $\mathcal{B}$ has a different interpretation. In fact, the coefficients are no more linked to the loading of the network risk but rather to the effect a given asset is causing to other assets or to its contribution to the network risk.

4.5 On the sign of the ρ coefficient

Up to this point, we have not yet discussed the sign of the ρ coefficient. Intuitively, we expect that the assets are positively related one to the other, as the links are coming from a network. We thus imagine that shocks transmit to connected assets preserving their sign. If we take simplified model with one single ρ coefficient, it is highly improbable we will ever observe negative coefficients. In fact, a single coefficient represents a sort of average reaction of the asset to the shocks coming from neighbors.

However, in a model accounting for the heterogeneity of the reaction to the network exposure, negative asset-specific coefficients might appear. In other words, we cannot exclude a-priori that a shock in one asset lead to an apposite movement of a linked asset. We motivate such a finding by making a parallel with negative correlations. If two assets are negatively correlated, their joint introduction in a portfolio lead to a decrease of the overall variance as compared to the case in which only one of the two assets were present. In a factor model, negative correlations across asset returns can be motivated by loadings to the (same) common factors having different signs. In our framework, negative correlations across asset returns can emerge both in response to different sings in the factor loadings but also due to the presence of negative asset-specific reaction to the network exposure.

Consider the reduced form of our model as represented in equation (16). In this case, the innovation term has a non-diagonal covariance. Let's also assume that the spatial proximity matrix W is time invariant and thus the reduced form model has time invariant betas and homoskedastic innovations. If we estimate the reduced form model, the innovations could show evidence of non-null correlations, some of them being negative. They can be due both to the presence of opposite reaction to the common factors, whose coefficients have been estimated by a biased estimator (due to model misspecification), but also due to the presence of negative ρ_i coefficients.

In a general model with heteregenous asset reaction to the network exposure, the components II and IV in the risk decomposition we have previously introduced, can become negative. In such a case, the network exposure reduces risk, and this could also be seen as a kind of flight-to-safety effect: if shocks hit financial assets and then transmit to industrial pro-cyclical sectors, we cannot exclude that the anti-cyclical sectors will anyway suffer.

Within our model, negative ρ might thus exist, but how can we interpret them from a pricing perspective? We read them as evidences of risk absorption due to the network exposure. In fact, a negative ρ_i allows a reduction of the exposure of one asset to the common factors, since the $i-th$ component of the second term in equation (19) becomes negative. Risk absorption has consequences also to expected returns, leading to a reduction of the contribution of network exposure. In fact, also the $i-th$ component of the third term in equation (35) will become negative.

5 Model estimation

We have seen how to interpret the model parameters and how to derive from the models intuitive decomposition both on the returns as well as on the total risks. However, model parameters must be estimated and this poses relevant challenges. Let us report the simultaneous model equation

$$AR_t = \alpha + \bar{\beta} F_t + \eta_t. \tag{39}$$

As standard econometrics textbook reports, identification conditions are required to estimate the parameters of A, α, $\bar{\beta}$ and $\mathbb{V}\left[\eta_t\right]$. The simple order condition of identification requires that the model parameters must be less than the parameters we can recover from the reduced form specification. In fact, the latter can be estimated by least square methods, and structural parameters could be recovered thanks to their relation with reduced form parameters. The reduced form model is

$$R_t = \alpha^{\star} + \bar{\beta}^{\star} F_t + \epsilon_t. \tag{40}$$

suggesting we can consistently estimate $4K$ mean parameters plus $\frac{1}{2}K\left(K+1\right)$ covariance parameters. However, an unrestricted structural specification, despite having the same number of parameters in the covariance, has $4K + K^2$ mean parameters.

The presence of assets interconnections, summarized into a network, allows a sensible reduction of the number of parameters included in the matrix A. In fact, if we have asset-specific network exposures and a single network, we have only K parameters in A. However, this is not sufficient to achieve identification of the model remaining parameters, since the order condition is still not satisfied. Identification is obtained by imposing the diagonality of $\mathbb{V}\left[\eta_t\right]$. Such a choice, which is economically motivated, allows satisfying the standard order condition for identification.

Nevertheless, further constraints are generally required on the model parameters.

Starting from the spatial econometrics literature, that takes a scalar time invariant ρ coefficient and a time invariant row-normalized W matrix, we must impose that $\frac{1}{\lambda_{min}} < \rho < \frac{1}{\lambda_{max}}$ where λ_{min} and λ_{max} are, respectively, the minimum and maximum eigenvalues of W. This constraint ensures the non-singularity of $I - \rho W$.

In our framework we deviate from traditional approaches in several ways. We first consider the case of a time-varying spatial matrix, that is W_t. A sufficient condition for the invertibility of $I - \rho W_t$ for all t is stated in the following assumption

Assumption 5.1. *The coefficient ρ satisfies the following condition*

$$\bar{\lambda}_{min}^{-1} < \rho < \bar{\lambda}_{max}^{-1} \tag{41}$$

where

$$\bar{\lambda}_{max} = \mathtt{min}\left\{\lambda_{t,max}\right\}_{t=1}^{T} \tag{42}$$

$$\bar{\lambda}_{min} = \mathtt{max}\left\{\lambda_{t,min}\right\}_{t=1}^{T} \tag{43}$$

and $\lambda_{t,max}$ and $\lambda_{t,min}$ are, respectively, the minimum and maximum eigenvalues of a matrix W_t. ■

If we have a diagonal matrix $\mathcal{R}$ containing the asset-specific reaction to the spatial links, we assume the non-singularity which is then validated in the estimation step of the model:

Assumption 5.2. *The diagonal coefficient matrix $\mathcal{R}$ is such that*

$$I - \mathcal{R}W_t \tag{44}$$

is non-singular for each matrix W_t. ■

Note that the previous assumption covers both the case of a time-invariant and time-varying spatial matrix. We further note that, when we consider a model with $\mathcal{R}$, we must impose an additional identification condition

Assumption 5.3. *The diagonal coefficient matrix* $\mathcal{R} = diag\,(\rho_1, \rho_2, \ldots, \rho_K)$ *is such that* $\rho_j = 0$ *if the matrix* $W_j = \left[W'_{j,1} W'_{j,2} \ldots W'_{j,T}\right]$, *with* $W_{j,t}$ *being the* $j-th$ *row of* W_t, *has non-null rank.*■

The previous assumption requires, irrespective of the number of W_t matrices, that if the $j-th$ row of all the matrices W_t contains only zeros (that is the asset j is not linked to any other asset in the various evolution of the network), then the asset j network impact coefficient is restricted to zero as it cannot be identified. This condition ensures that the asset specific impact to the network links is estimated only if such link exist for at least one point in time.

Finally, we note that, if we introduce many spatial matrices, a further identification condition is required. Let us assume the presence of m different time-invariant networks that could have either a constant impact (with scalar ρ coefficients) or heterogeneous impacts (with m matrices $\mathcal{R}$. We impose the following:

> **Assumption 5.4.** *[i] If the matrix* A *satisfies* $A = I - \sum_{l=1}^{m} \rho_l W_j$, *we assume* A *is invertible. [ii] If the matrix* A *satisfies* $A = I - \sum_{l=1}^{m} \mathcal{R}_l W_l$, *we assume* A *is invertible and the matrices* $\mathcal{W}_j = \left[W'_{j,1} W'_{j,2} \ldots W'_{j,m}\right]$, *with* $W_{j,i}$ *being the* $j-th$ *row of matrix* W_i, *have all full column rank.*■

The second item of the previous assumption requires that all the network impact coefficients are identified. Otherwise, zero restrictions must be imposed on the matrices $\mathcal{R}_l$.

The use of covariance restrictions has a consequence for the estimation of model parameters. In fact, those must be jointly evaluated, despite the linear model structure might allow for single equation (single asset) parameter estimation.

Under the two strong parametric restrictions we impose (the structure on A and the absence of correlation across the idiosyncratic shocks), a viable approach is that of Full Information Maximum Likelihood (FIML) methods. However, if K is even moderately large, the total number of parameters to be estimated in the restricted structural model,

$7K$, might be quite large. Fortunately, we can follow the approaches commonly used in spatial econometrics, namely the use of concentrated likelihoods. As in Elhorst (2003), and LeSage and Pace (2009), we start by writing the full model log-likelihood

$$\begin{aligned} L(\Theta) &= \sum_{j=1}^{T} l_t(\Theta), & (45) \\ l_t(\Theta) &\propto -\frac{1}{2}log|\Omega| - \frac{1}{2}e_t'\Omega^{-1}e_t, & (46) \\ e_t &= R_t - \bar{\alpha} - \mathcal{R}WR_t - \bar{\beta}F_t. & (47) \end{aligned}$$

where Ω is a diagonal matrix. We can note that, if ρ is known, we can write

$$R_t - \mathcal{R}WR_t = Z_t = \bar{\alpha} + \bar{\beta}F_t + \varepsilon_t \tag{48}$$

Therefore, with a know network exposure parameter matrix $\mathcal{R}$, we might estimate the parameters in $\bar{\alpha}$ and in $\bar{\beta}$ by least square methods, obtaining the well-known expressions. In addition, we might even recover standard estimators for the innovation variance. This suggests that the network exposure parameters can be easily obtained by maximizing the concentrated likelihood obtained by replacing the other parameters by their least square estimators.

This will be of a relevant computational importance as it allows reducing the parameters to be jointly estimated to $2K$ if we concentrate the likelihood with respect to $\bar{\alpha}$ and $\bar{\beta}$, and to K if we concentrate also with respect to the innovation variance. Standard errors can be recovered from the full-model likelihood by numerical evaluations of the Hessian (and of the gradient if we take a robust parameters covariance matrix). Note that this approach can be followed even if the spatial matrix W is time-varying, or with zero restrictions added to specific parameters of $\mathcal{R}$ to ensure model identification.

6 Simulation Analysis

To show the capabilities of the proposed framework and to underline the effect due to model misspecification, that is, neglecting the network links across assets, we include in this section a set of simulations.

6.1 Scalar network impact

At first, we concentrate on the simplest model, with time invariant W and scalar ρ. Such a baseline design will provide some expected results, as we will point out in few lines. The first data generating process we consider is a linear factor model with a unique risk factor, a scalar network impact and a fixed (and known a-priori) network matrix W:

$$(I - \rho W)(R_t - \mathbb{E}[R_t]) = \bar{\beta} F_t + \eta_t, \tag{49}$$

with the following specification for parameters, shocks and asset interconnections:

- We consider $K = 100$ assets, thus focusing on a somewhat large cross-sectional dimension, and assume we simulate monthly returns;
- The ρ coefficient assumes fixed values $\rho \in \{0, 0.25, 0.5, 0.75\}$ allowing to compare the case of no network impact with different and increasing levels of network impact; note that when $\rho = 0$ our model collapses on the traditional linear factor model;
- The factor loading coefficients are randomly generated from $\beta_i \sim \mathcal{U}(0.8, 1.2)$, $\quad i = 1, 2, \ldots, K$ with thus positive factor loadings with an average value of 1;
- We simulate the factor returns from a Gaussian density, $F_t \sim \mathcal{N}(\mu_F, \sigma_F^2)$ with $\mu_F = 0$ and $\sigma_F = 15\%$ on a yearly basis;
- The risky assets expected return equal $\mathbb{E}[R_t] = r_f + (I - \rho W)^{-1} \beta \Lambda$ with β being the $K-$dimensional vector of betas simulated above, the factor risk premium equals 5% on a yearly basis, and the risk-free rate is set to 1% on a yearly basis;

T	$\rho = 0$		$\rho = 0.25$		$\rho = 0.5$		$\rho = 0.75$	
	Mean	Std.dev	Mean	Std.dev	Mean	Std.dev	Mean	Std.dev
				Distortions for ρ				
200	0.067	0.084	0.062	0.075	0.050	0.058	0.028	0.032
500	0.029	0.040	0.028	0.037	0.024	0.030	0.014	0.017
1000	0.015	0.024	0.014	0.023	0.012	0.019	0.007	0.011
				Cross-sectional average of the β distortions				
200	-0.072	0.091	-0.088	0.107	-0.096	0.107	-0.119	0.133
500	-0.032	0.043	-0.040	0.053	-0.045	0.061	-0.059	0.072
1000	-0.016	0.026	-0.020	0.032	-0.022	0.034	-0.031	0.045

Table 1: Mean and standard deviation for the ρ and β distortions under correct model specification across different values of the network impact and different sample sizes. Values computed across 500 replications.

- The W matrix comes from a simple and naive design: each of its off-diagonal elements is extracted from a Bernoulli density $w_{i,j} \sim \mathcal{B}(p_B)$ with $p_B = 0.3$; the simulated W is then row-normalized;
- The shocks are extracted from a Gaussian $\eta_t \sim \mathcal{N}(0, \Omega)$, with Ω being a diagonal matrix with diagonal elements extracted from a uniform, $\omega_{i,i}^{\frac{1}{2}} \sim \mathcal{U}(10\%, 25\%)$ with limits referring to a yearly horizon;
- We simulate 500 sequences of monthly returns with three different sample sizes, $T = 200, 500, 1000$.

The baseline simulation provides expected results. Firstly, the estimators of the ρ coefficients and of the (structural) β vector have an asymptotically normal density with dispersion decreasing with the sample size, see Table (1). Figure (1) report a kernel estimate of the distortion $\hat{\rho} - \rho$ across different values of ρ, while Figure (2) provides a kernel density for the cross-sectional average (over assets) of the distortions $\hat{\beta}_i - \beta_i, \quad i = 1, 2, \ldots, K$; all graphs contain the plots for the three different sample sizes. We note that the coefficients converge to the true values and that their dispersion decreases with the sample size, as expected.

If we estimate a standard linear factor model on the series simulated from (49), that is we fit

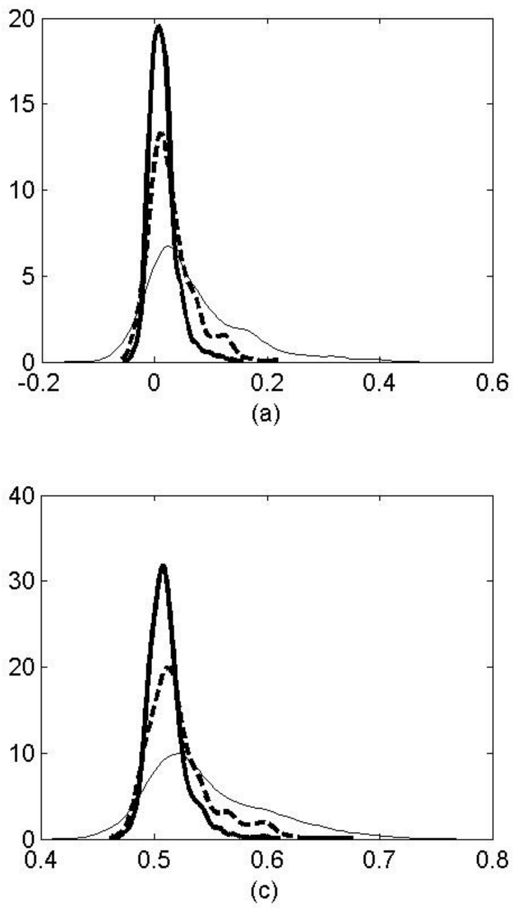

Figure 1: Distortions of the ρ coefficients under the correctly specified model. True values: (a) $\rho = 0$, (b) $\rho = 0.25$, (c) $\rho = 0.5$, and (d) $\rho = 0.75$. Lines refer to different sample sizes, $T = 200$ thin grey line, $T = 500$ dashed line, and $T = 1000$ thick black line.

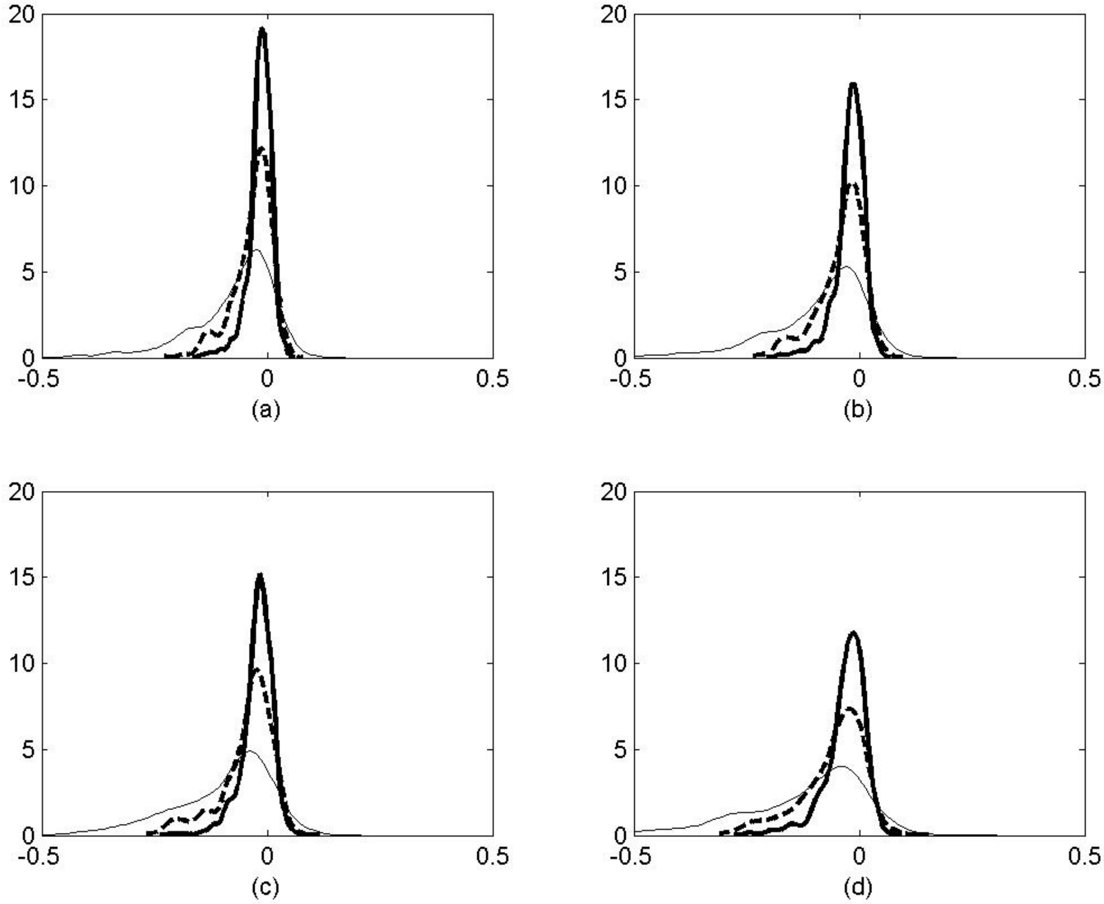

Figure 2: Cross-sectional average of the distortions for the β coefficients under the correctly specified model across different ρ values. True values: (a) $\rho = 0$, (b) $\rho = 0.25$, (c) $\rho = 0.5$, and (d) $\rho = 0.75$. Lines refer to different sample sizes, $T = 200$ thin grey line, $T = 500$ dashed line, and $T = 1000$ thick black line.

T	$\rho = 0$		$\rho = 0.25$		$\rho = 0.5$		$\rho = 0.75$	
	Mean	Std.dev	Mean	Std.dev	Mean	Std.dev	Mean	Std.dev
	Cross-sectional average of the distortions $\hat{\gamma}_1 - \beta$							
200	0.000	0.026	0.337	0.027	1.013	0.028	3.033	0.043
500	0.000	0.016	0.337	0.017	1.011	0.018	3.034	0.027
1000	0.000	0.011	0.337	0.012	1.012	0.012	3.035	0.019
	Average residual correlations under the misspecified linear factor model							
200	0.000	0.003	0.009	0.007	0.034	0.014	0.149	0.035
500	0.000	0.002	0.009	0.007	0.033	0.013	0.149	0.035
1000	0.000	0.001	0.009	0.006	0.033	0.013	0.149	0.035
	Average residual correlations under the correctly specified model							
200	-0.001	0.004	-0.002	0.003	-0.002	0.003	-0.002	0.003
500	-0.001	0.002	-0.001	0.002	-0.001	0.002	-0.001	0.002
1000	0.000	0.001	0.000	0.001	0.000	0.001	-0.001	0.001

Table 2: Mean and standard deviation for the cross-sectional average of the distortions $\hat{\gamma}_1 - \beta$ under model misspecification, upper panel; average residual correlation under model misspecification, central panel, and under correct model specification, lower panel. Statistics computed across different values of the network impact and different sample sizes. Values computed across 500 replications.

$$R_t = \gamma_0 + \gamma_1 F_t + \varepsilon_t \tag{50}$$

we have that $\gamma_0 = \mathbb{E}[R_t]$, $\gamma_1 = (I - \rho W)\beta$, and $\mathbb{V}[\varepsilon_t] = (I - \rho W)^{-1} \Omega (I - \rho W')^{-1}$. Therefore, estimating the linear factor model we estimate the reduced form representation of our model with network dependence. The γ_1 coefficients, by construction, will be larger than the structural coefficients β when we simulated from a data generating process with positive ρ. This is confirmed by Figure (3) and Table (2) where we report the kernel density for the cross-sectional average of $\hat{\gamma}_{1,i} - \beta_i, \quad i = 1, 2, \ldots K$ and some descriptive statistics. Moreover, the residuals of the linear factor model will be correlated, with average correlation increasing with ρ, see Table (2).

Figure (3) and Table (2) confirm that by fitting a linear factor model we estimate a beta much larger than the structural value, with distortion increasing with the impact coming from network connections. As a consequence, the value of the true and structural factor loading might sensibly differ from the one empirically observed, being doubled for

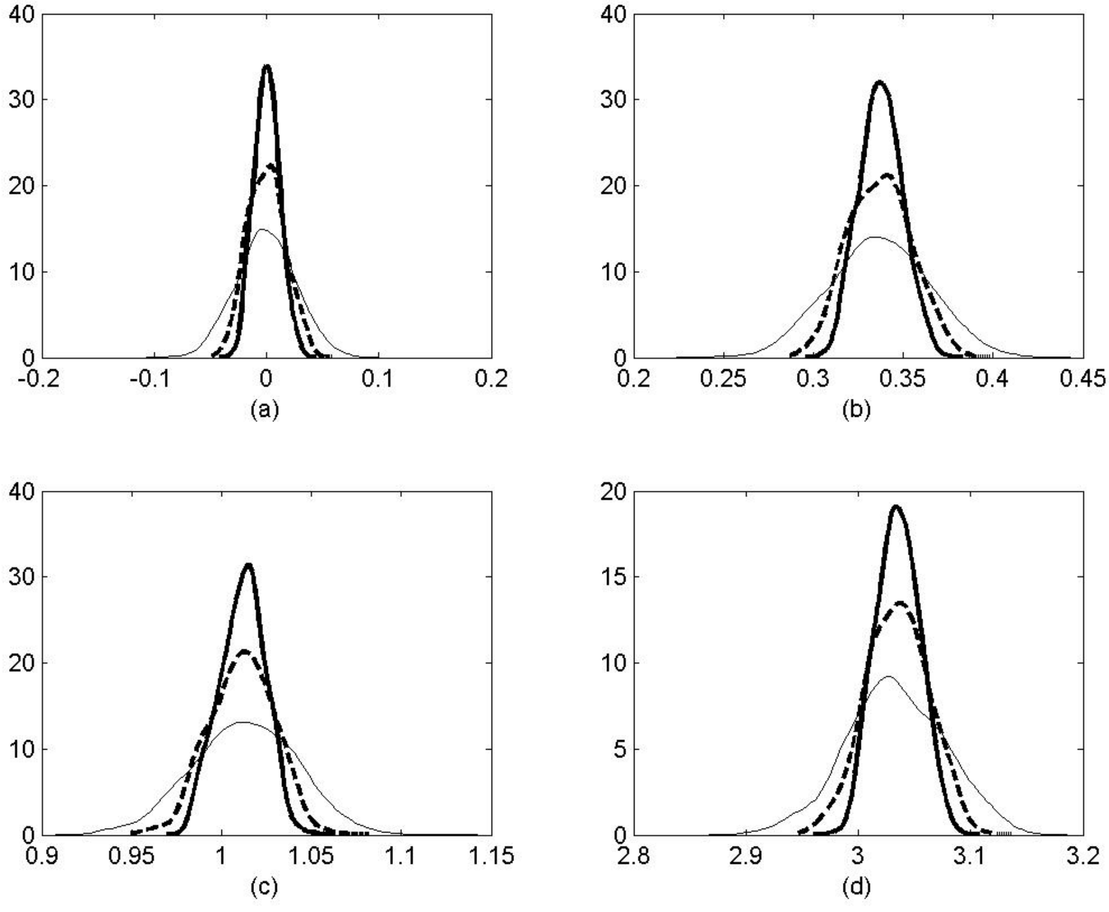

Figure 3: Cross-sectional average of the distortions $\hat{\gamma}_1 - \beta$ under the misspecified model across different ρ values for the data generating process. True values: (a) $\rho = 0$, (b) $\rho = 0.25$, (c) $\rho = 0.5$, and (d) $\rho = 0.75$. Lines refer to different sample sizes, $T = 200$ thin grey line, $T = 500$ dashed line, and $T = 1000$ thick black line.

ρ values equal to 0.5, thus not particularly elevate.

When analyzing the residuals correlations, we see that they are zero when the linear factor model is correctly specified, that is when $\rho = 0$. However, in the presence of network impact, the residual correlations start drifting away from zero, with values increasing with ρ. On the contrary, under correct model specification, the residual correlations are almost zero, as expected.

We move then to the estimation of the factor risk premium. We adopt the widely used two-pass regression approach of Black et al. (1972) and Fama and McBeth (1973). In linear factor models the first stage corresponds to the estimation of the factor loadings, that is the betas. Differently, in our model the first stage equals the estimation of reduced form betas starting from the estimated ρ coefficient and corresponds to a by-product of the concentrated maximum likelihood estimation approach adopted. We stress that, under scalar ρ and with a static W, the reduced form betas and the linear factor model betas are asymptotically equivalent. The second regression is a cross-sectional one, takes as dependent the average risky asset returns and regress them on the estimated betas (reduced form betas in our model). As pointed out by Black et al. (1972) the estimated risk premium suffer for an error-in-variable problem and is thus inconsistent. Standard solutions include: grouping assets into portfolios, increasing the sample size, increasing the cross-sectional dimension. We take the second one, since we are working in a purely simulation setting where we do not control for risky asset *market value*. As a consequence, we expect distortions in the estimation of the risk premiums for short sample sizes, and, given the asymptotic equivalence of the betas no difference between our model and the misspecified linear factor model. However, such an expected result is not impacting on the purpose of our simulation design as our final objective is not the correct estimation of the risk premiums but rather highlighting the differences in the estimated risk premiums obtained from either a correctly specified model or a misspecified linear factor model. We finally point out that the cross-sectional estimation of the risk premium could come from either a standard OLS as well as a GLS estimator. For the latter, we note that the

T	$\rho = 0$		$\rho = 0.25$		$\rho = 0.5$		$\rho = 0.75$	
	Mean	Std.dev	Mean	Std.dev	Mean	Std.dev	Mean	Std.dev
Estimated risk premiums from a linear factor model								
200	0.415	0.317	0.417	0.318	0.418	0.318	0.419	0.319
500	0.417	0.198	0.418	0.199	0.418	0.199	0.418	0.199
1000	0.423	0.137	0.423	0.138	0.423	0.138	0.423	0.138
Estimated risk premiums from a correctly specified model								
200	0.418	0.322	0.419	0.323	0.420	0.324	0.421	0.325
500	0.418	0.200	0.419	0.200	0.419	0.200	0.419	0.200
1000	0.423	0.138	0.423	0.138	0.424	0.138	0.424	0.138

Table 3: Mean and standard deviation of the estimated risk premiums across the 500 replications. The cross-sectional regression adopts an OLS estimator. The true risk-premium corresponds to 0.4167 at the monthly frequency.

correct model specification allows for a more precise design of the residuals covariance (in the reduced form representation of our model).

We report in Table (3) the estimated risk premiums. As expected, the premiums are very close to the true value with a dispersion decreasing in T. The limited distortions depend on the large sample sizes we consider.[10] No difference emerge by comparing the correctly and incorrectly specified models. Finally, we point out that both the OLS and GLS estimators provide substantially equivalent results, and thus we reported only the OLS case.

As a further example, we consider the $1/N$ portfolio variance, concentrating on the role played by the idiosyncratic risks. We order assets on the basis of their idiosyncratic risk and decompose the portfolio idiosyncratic risk into the structural component and the network effect. We consider portfolios with N varying from 5 to 100. Figure 4 reports the decomposition both in absolute and relative terms. Notably, the impact of network exposure induces a decrease of the idiosyncratic risks much smaller than the one associated solely on the structural risks, and with a relative weight increasing over time. Such a result lead to diversification benefits that are reduced compared to the ideal case of independent idiosyncratic shocks (associated with the reduced-form model representation).

[10]Similar results have been obtained with shorter samples of 60 and 120 observations.

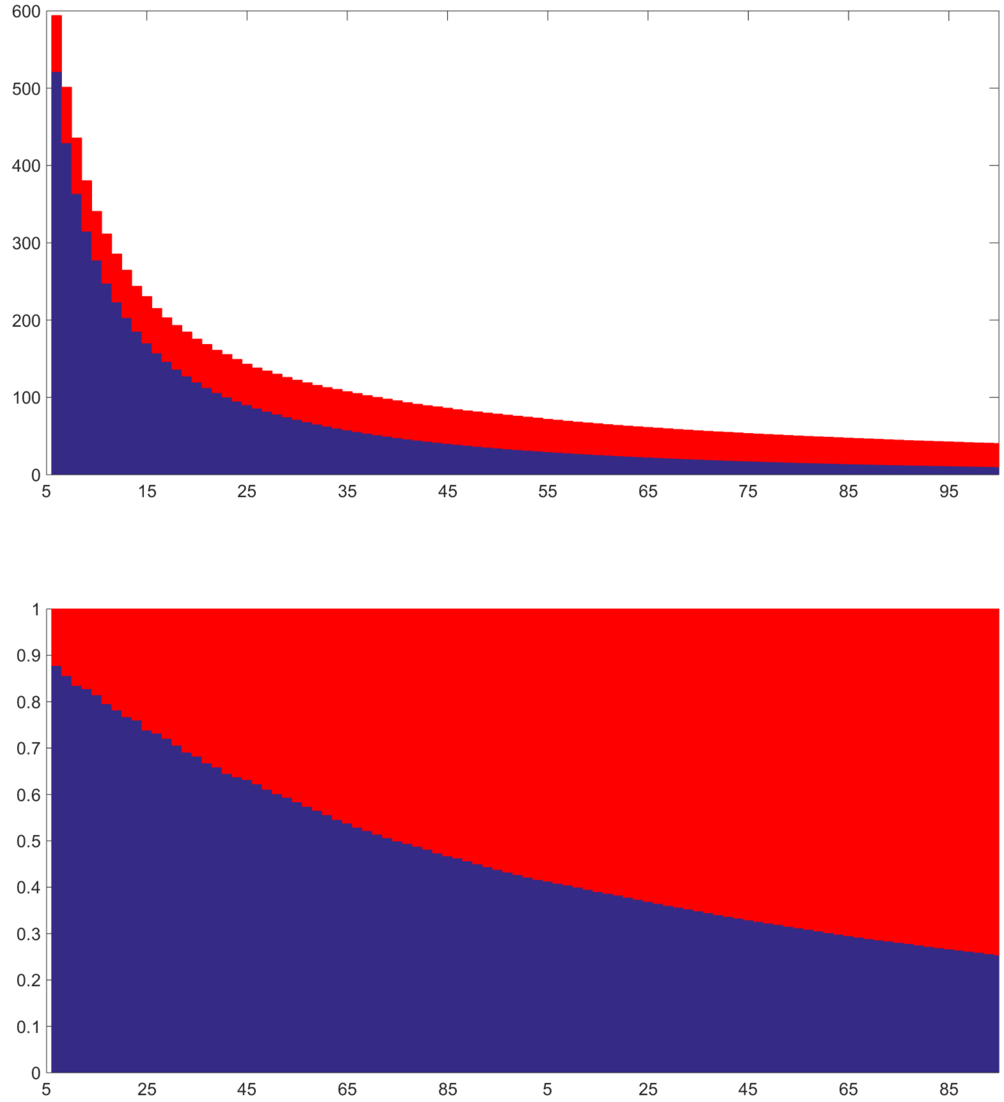

Figure 4: $1/N$ portfolio idiosyncratic risk components: structural risk (blue) and network-related risk (red) across different portfolio sizes using the same assets adopted in the simulations and with $\rho = 0.5$. Absolute decomposition (upper) and relative decomposition (lower).

To evaluate the impact of the various settings of the data generating process, we run a number of robustness checks: we simulate the β vector from a Gaussian with mean 1 so that betas are more concentrated around the mean but also characterized by a larger variance; we increased the volatility of the common factor to a yearly value of 25%; we changed the network density by setting $p_B = 0.15$ and $p_B = 0.45$, or, maintaining the same density, we simulate different networks; we modified the factor risk premium to $\Lambda = 3\%$ or $\Lambda = 10\%$; we increased the relevance of the idiosyncratic shocks sampling elements of Ω as $\omega_{i,i}^{\frac{1}{2}} \sim \mathcal{U}\left(20\%, 50\%\right)$.

All these elements do not affect the previously reported results.[11]

6.2 Heterogeneous network impact

The second simulation design we consider adds the heterogeneity in the network impact. We thus move from the ρ coefficient to the diagonal matrix $\mathcal{R}$. The asset-specific network impact comes from a Normal density, $\rho_i \sim \mathcal{N}\left(0.5, 0.01\right)$, such that with probability close to 99% the ρ takes values between 0.25 and 0.75. In order to control the computational time, we reduce the cross sectional dimension for this simulation and set $K = 20$.

For that case, we provide in Figure (5) a kernel density for the cross sectional average of $\hat{\rho}_i - \rho_i, \quad i = 1, 2, \ldots, K$ for different sample sizes.

We do not provide further results for the estimated factor loadings and residual correlations associated with the fit of the standard linear factor model as they provide the same evidences as in the first simulation design: the betas are larger than the structural values and residuals are correlated. We only point out that, in the presence of heterogeneity in the network impact, residuals correlations are even higher than in the case of scalar ρ.[12]

Differently, we provide in Table (4) further evidences from the risk premium estimation. Notably, the estimated risk premiums present some slight distortion (overestimation) as opposed to the previous simulation design. We link them to the introduction of the asset heterogeneous impact of the network, that intuitively amplifies the impact of the

[11] Additional figures and tables are available upon request.

[12] Additional tables and figures are available upon request.

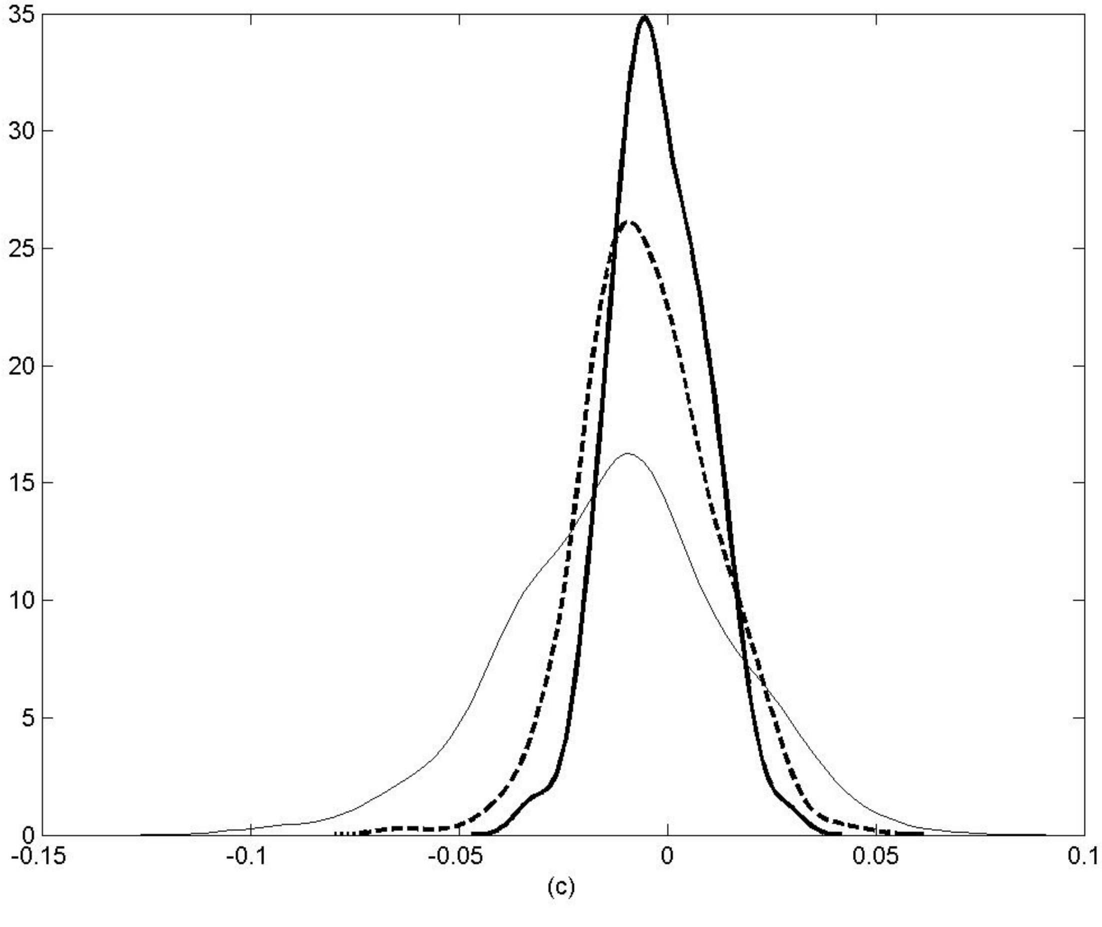

Figure 5: Distortions of the cross-sectional average of $diag\,(\mathcal{R})$ under the correctly specified model. Lines refer to different sample sizes, $T = 200$ thin grey line, $T = 500$ dashed line, and $T = 1000$ thick black line.

	Misspecified model				Correctly specified model			
T	OLS		GLS		OLS		GLS	
	Mean	Std.dev	Mean	Std.dev	Mean	Std.dev	Mean	Std.dev
200	0.440	0.326	0.440	0.324	0.443	0.331	0.442	0.328
500	0.443	0.201	0.443	0.200	0.444	0.202	0.444	0.201
1000	0.429	0.149	0.429	0.148	0.429	0.149	0.430	0.147

Table 4: Mean and standard deviation of the estimated risk premiums across the 500 replications. The cross-sectional regression adopts an OLS or GLS estimator. The true risk-premium corresponds to 0.4167 at the monthly frequency.

error-in-variable problem. Increasing the sample size, the distortions tend to decrease as well as the dispersion of the estimated risk premiums. There are no differences by contrasting the two estimation approaches, as in the previous case. Finally, as expected, the correctly specified model and the misspecified model provide comparable results. We stress this is a consequence of the data generating process we consider, where the risk premium is estimated by looking at the reduced form betas. In the current DGP, with heterogeneous network impacts, the linear factor model provides consistent estimates of the reduced form betas, but does not allow separating the network and structural elements affecting the betas.

For the first design we provided an example associated with the decomposition of the $1/N$ portfolio idiosyncratic risk into the standard component and network-related component. We repeat here the same exercise with two different $\mathcal{R}$ matrices: the first is the one used above, while the second allows also for the presence of negative ρ_i coefficients in half of the simulated assets. This second example allows highlighting the *risk absorption* effect of the network exposure. While for the first case results are qualitatively similar to those of the scalar ρ case. Differently, when we introduce negate ρ_i values, and order assets with respect to their ρ_i value (in a descending order), we note that the introduction of assets with negative ρ_i lead to a sensible decrease of the interconnection impact on the idiosyncratic risk (the fourth component of the variance decomposition). Such an effect could even become negative, thus leading of the absorption of risk by the linked assets, or, in other words, as the amplification of the diversification benefits. This is evident

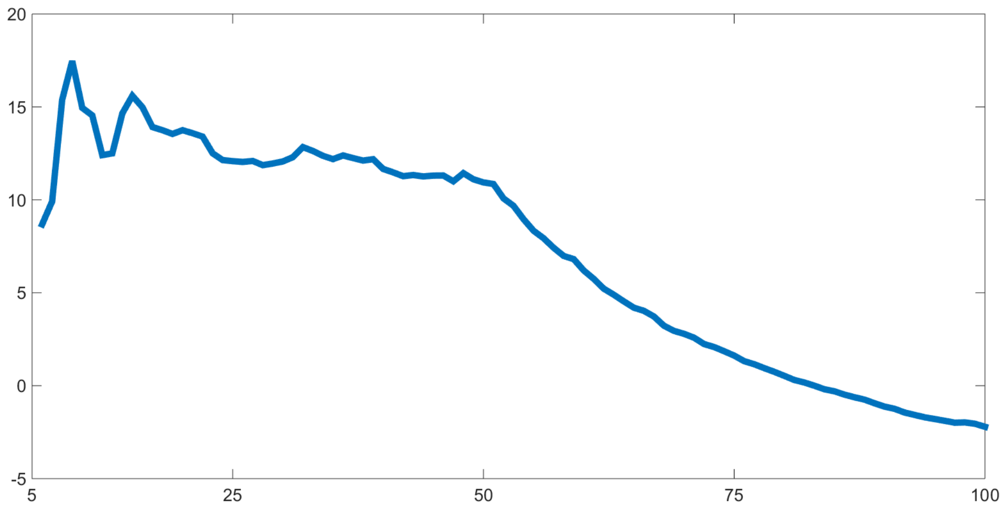

Figure 6: Fourth component of the $1/N$ portfolio variance - impact of interconnections on the idiosyncratic risk. Assets are ordered in a descending way over ρ_i values.

in Figure (6) where we report the contribution of the fourth component to the $1/N$ portfolio variance, where the portfolio size increases from 5 to 100 assets, and assets have a descending order on ρ_i. The last 50 assets have negative network impact, and the contribution of the interconnections on the idiosyncratic risk becomes negative around asset 80.

6.3 Dynamic W and heterogeneous network impact

In the third simulation design we combine the heterogeneity in the asset network impact with the time-change in the network connections across assets. Now the data generating process is

$$(I - \mathcal{R}W_t)(R_t - \mathbb{E}[R_t]) = \bar{\beta}F_t + \eta_t, \tag{51}$$

Note that, differently from the previous designs, the expected returns, conditional to W_t, are dynamic. To generate a time-change in the W_t we chose a simple approach, starting from the empirical evidence that the links across assets are persistent, that is, we do not have networks completely different at time t and at time $t+1$. Moreover, as commented in the previous section, we do not allow for a change in W_t at every t but rather modify W_t every $m = 20$ observations.

We change W_t according to the following scheme: at time 1 we sample W_1 as in the first design, that is the off-diagonal elements $w_{i,j} \sim \mathcal{B}(p_B)$ with $p_B = 0.3$; every m observations, each off diagonal $w_{i,j}$, can take only two values, 0 or 1 and is driven by a Markov chain with diagonal elements of the transition matrix set as $p_{00} = p_{11} = 0.9$. Such a choice ensures persistence in the W_t with possible long-lasting increases/decreases in the associated network density. Finally, we point out that the W_t matrices have been normalized with the maximum row normalization.

We now provide a number of results recovered from this simulation design. At first, we focus on the coefficients ρ_i. As in the previous case, Figure (7) reports the kernel density for the average of $diag\left(\hat{\mathcal{R}}\right) - diag\left(\mathcal{R}\right)$ for different sample sizes. We observe a convergence (on average) of the concentrated estimates to the true values with increasing sample sizes, as expected.[13]

Secondly, we note that, with the data generating process in (51), the linear factor model does not estimate the reduced from betas as, by construction, those are time-varying $\gamma_1 \neq \left(I - \mathcal{R}W_t\right)^{-1}\beta$. Therefore, to evaluate the distance between those two values, we compute the distortions $\hat{\gamma}_1 - \left(I - \mathcal{R}W_t\right)^{-1}\beta$ and compare them with the distortions under the correctly specified model $\left(I - \hat{\mathcal{R}}W_t\right)^{-1}\hat{\beta} - \left(I - \mathcal{R}W_t\right)^{-1}\beta$; in both cases, we focus on the cross-sectional averages of the distortions. We collect results in Table (5). From the table it clearly emerges that the correctly specified model captures the evolution of the reduced form betas which, we remind, are conditional to the knowledge of the network links. Moreover, the distortions decreases both in mean and in their dispersion. Differently, for the misspecified model the distortions do not clearly converge toward the true values but seems to be characterized by an average overestimation of the factor impact.

[13]Detailed tables with coefficient-specific results are available upon request.

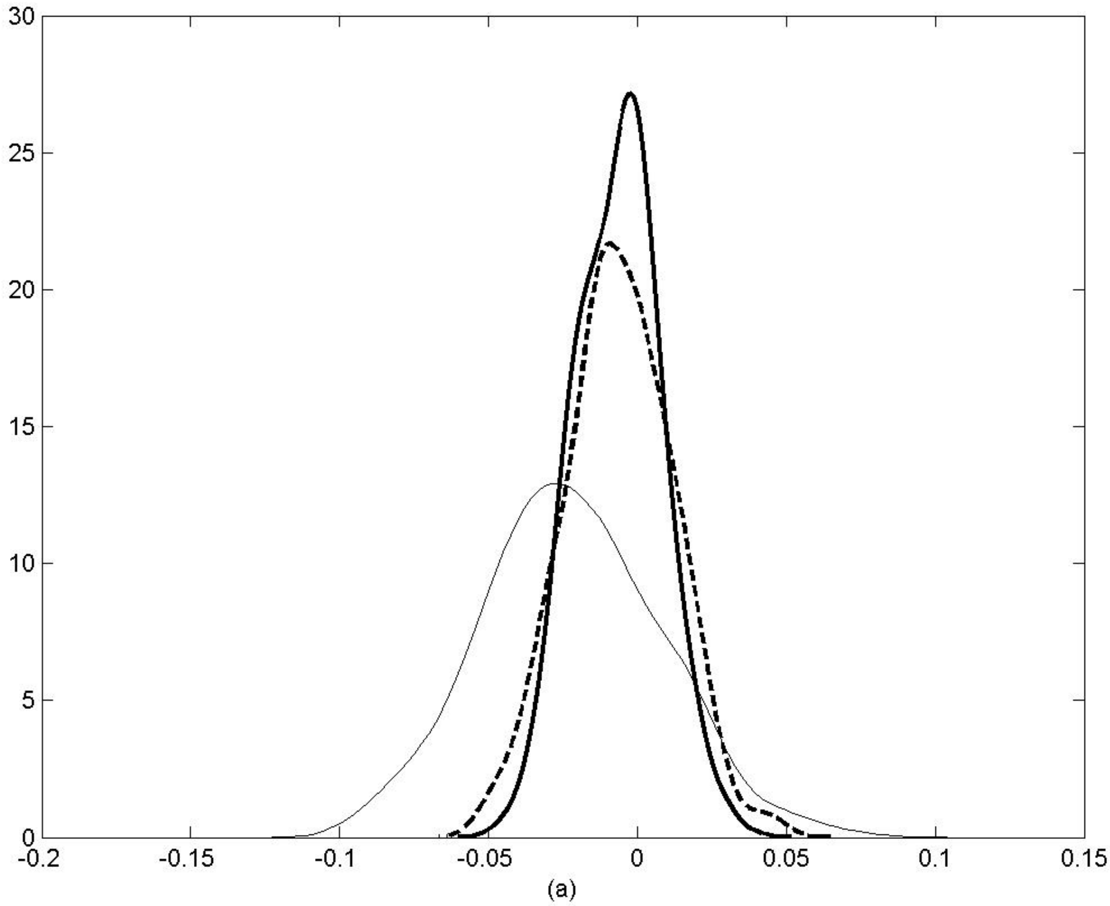

Figure 7: Distortions of the cross-sectional average of $diag\,(\mathcal{R})$ under the correctly specified model. Lines refer to different sample sizes, $T = 200$ thin grey line, $T = 500$ dashed line, and $T = 1000$ thick black line.

T	Misspecified model		Correctly specified model	
	Mean	Std.dev	Mean	Std.dev
200	0.153	0.008	0.082	0.014
500	0.205	0.003	0.055	0.008
1000	0.194	0.002	0.040	0.006

Table 5: Mean and standard deviation of the cross sectional averages for the distortions between the estimated betas under the misspecified linear factor model and the reduced form betas induced by the true model (left columns), and between estimated and true reduced form betas under the correctly specified model (right columns).

	Misspecified model				Correctly specified model			
T	OLS		GLS		OLS		GLS	
	Mean	Std.dev	Mean	Std.dev	Mean	Std.dev	Mean	Std.dev
200	0.413	0.316	0.412	0.314	0.417	0.323	0.415	0.355
500	0.417	0.194	0.418	0.193	0.418	0.200	0.415	0.217
1000	0.417	0.146	0.417	0.146	0.419	0.149	0.417	0.157

Table 6: Mean and standard deviation of the estimated risk premiums across the 500 replications. The cross-sectional regression adopts an OLS or GLS estimator. The true risk-premium corresponds to 0.4167 at the monthly frequency.

Finally, we move to the risk-premium estimation and report in Table (6). In that case, the GLS estimator we adopted for the correctly specified model takes into account the known covariance structure across across the reduced form residuals (and thus account for an impact of the network links in the estimation of the residual covariance). We first highlight that the OLS and GLS estimates are substantially equivalent, thus there is no effect associated with the estimator adopted. Then, we come to the most interesting finding: the risk premium are very close to the true values for both the correctly and incorrectly specified models, and similarly the risk premium dispersions are very close under the two estimated models. Distortions were somewhat expected, however, they have been canceled out by two elements. Firstly, by the introduction of an averaging across the different W_t matrices. In fact, under the linear factor model we estimate the betas using the entire sample size, implicitly being affected by the various networks. The reduced form model estimators are implicitly averaging across the W_t. Secondly, by the pattern characterizing the W_t matrix, which is not exploding. Nevertheless, this second element has a minor role. Further simulations with different dynamic for the W_t introducing a linear or exponential increase in the network density, or a level shift in the network density, confirm the finding.[14]

[14] Additional results are available upon request.

7 Conclusions

Part of the literature postulates that systemic risk is strictly related (if not equal) to systematic risk. In this paper we elaborate on this hypothesis and introduce a modelling framework where systemic and systematic risks co-exist. The model is a variation of the traditional CAPM where networks are used to infer the exogenous/lagged and contemporaneous links across assets. We show that this approach allows us to decompose the risk of a single assets (or a portfolio) in four components: the two classical systematic and idiosyncratic components and (i) the impact of the asset interconnections on the systematic risk component, that is the contribution of network exposure to the systematic risk component and (ii) the effect of interconnections on the idiosyncratic risk on the systematic risk component, that is the amplification of idiosyncratic risks that generates systematic/non diversifiable risk. Our approach allows us also to decompose the risk premium component of returns in three components: the risk premium associated with (i) common factors exposures, (ii) impact of asset connections to common factors, and (iii) the amplification effects of idiosyncratic risk. The simulation analysis we perform shows that the new model we propose can be used to analyze in detail which implications different notions of systemic risk have for equilibrium stock returns and volatilities and analyze how similar to or different from exposure to common factors systemic risk actually is. This new model is relevant for policy makers and regulators, since they need to be aware of the implications of the different possible policy choices on network connections and their effects on equilibrium stock returns and volatilities, as well as to investors and other market participants, since they need to understand if and to what degree systemic risk network connectivity has an impact on risk premia, volatilities, and spillovers between markets. The model could be analyzed not only through simulations but also on real data. We plan to work on this in our next project.

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